Extensive games

Theory of Individual and Strategic Decisions

MSc Human Decision Science

Maastricht University

- **Strategic games**: actions are taken simultaneously at once
- **Extensive games**: actions are taken sequentially over time

Example (Chain-store paradox (Selten, 1976))

Ann owns a store in a small town, and Bob is the owner of a big chain considering whether to open a shop in the same town. If he opens a shop, Ann can decide whether to fight him (through a price war) or to share the market profits (through collusion):

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Share	\$50 , \$5	\$25 , \$25
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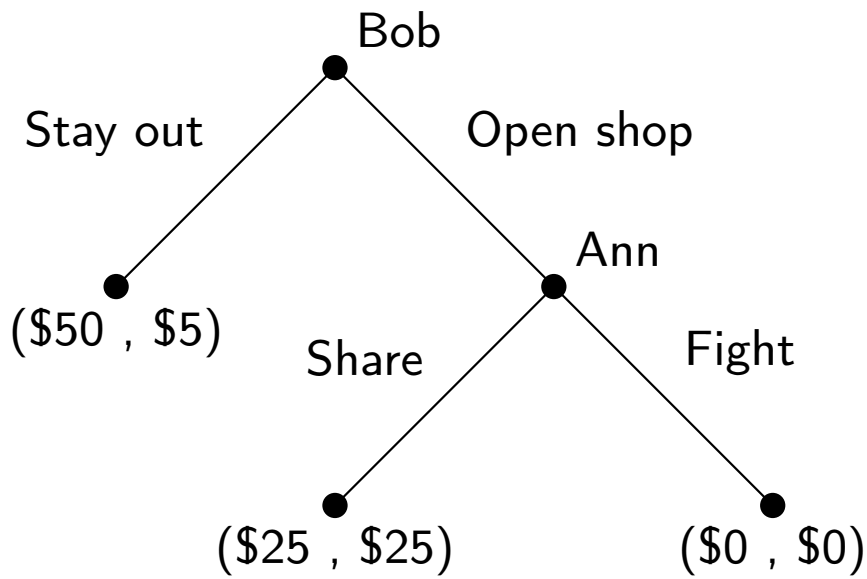

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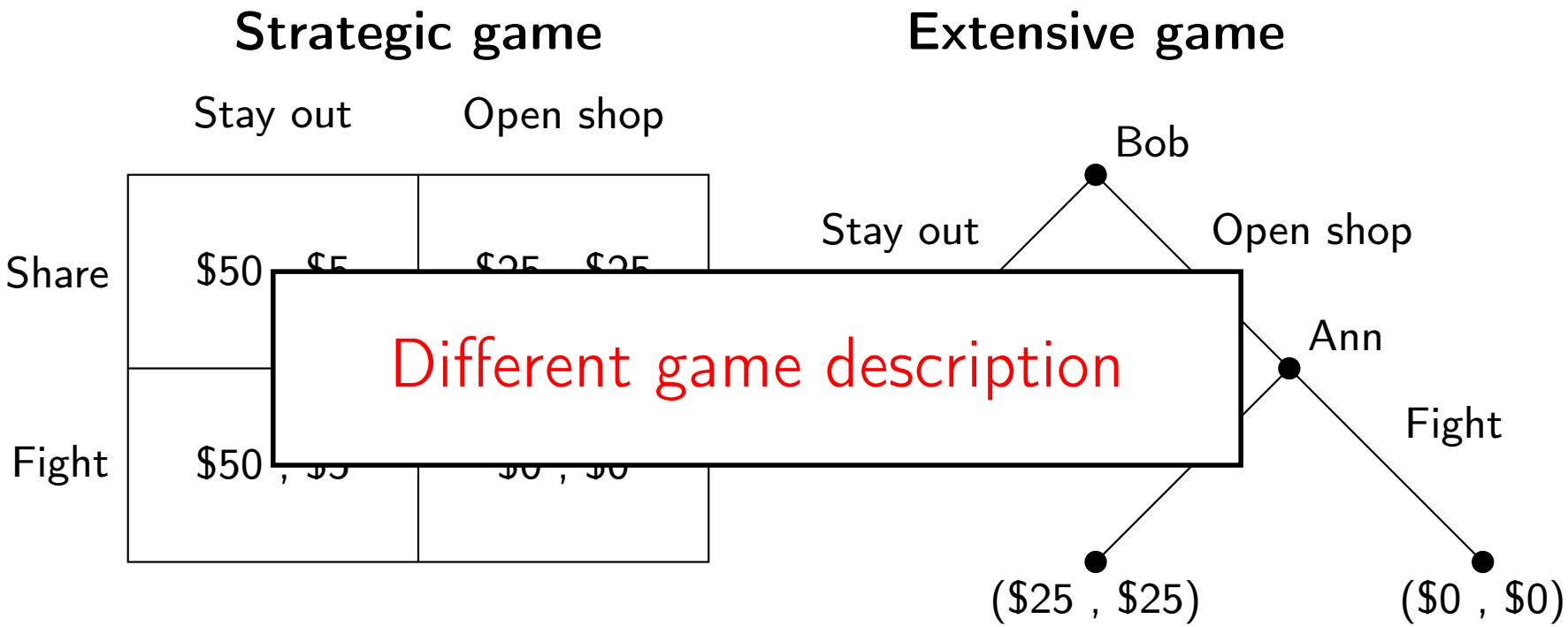

- **Game description** contains two parts:
  1. The players $N = \{1, \ldots, n\}$
  2. The game tree describes the sequence of actions for each player
  3. One outcome for each sequence of actions
  4. Preferences $\succeq^i$ represented by a utility u^i over the set of outcomes for each player i
- Key difference to strategic games is the game tree (previously we simply had "available actions for each player")
- In this course we use no mixed strategies in extensive games

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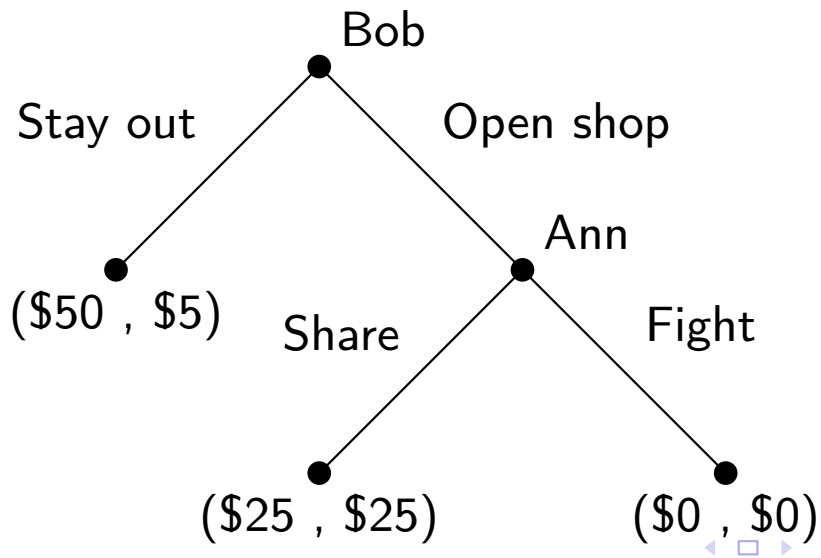

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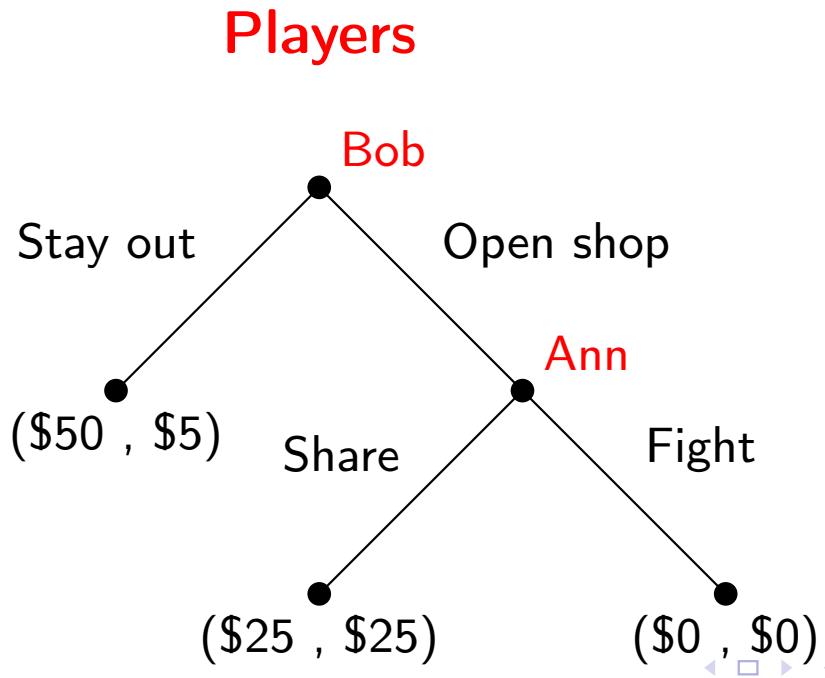

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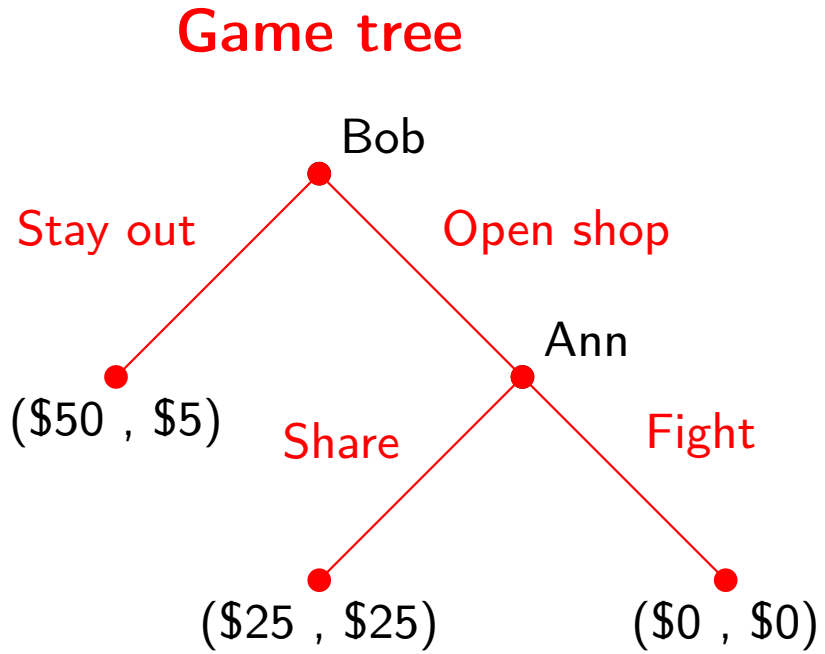

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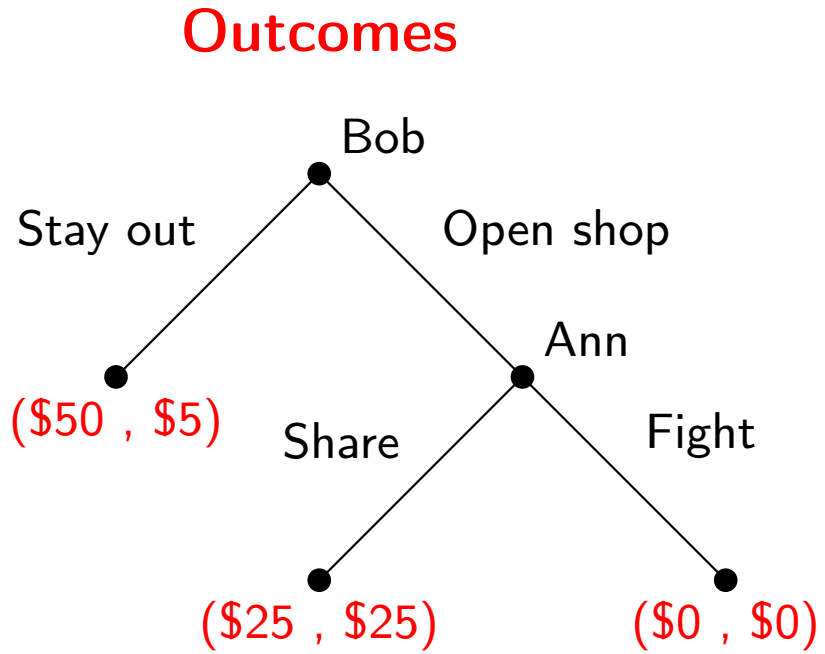

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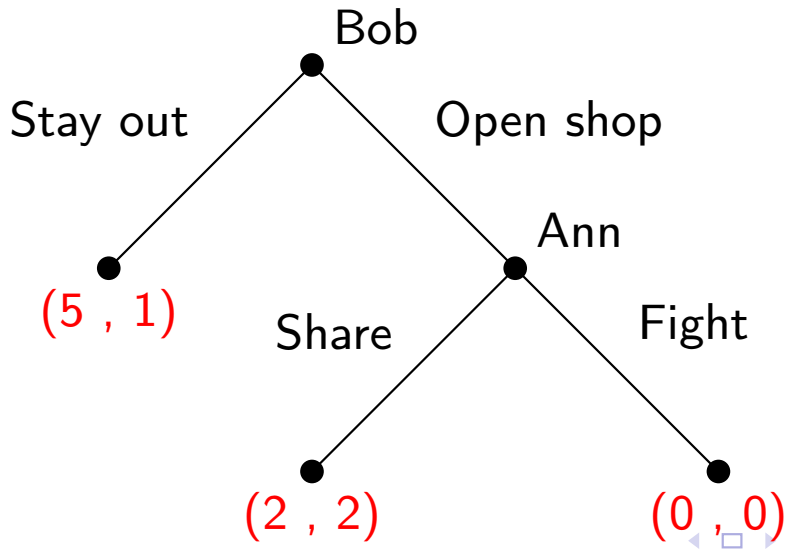
Preferences/Utilities



Some **terminology/notation** about game tree:

- **Nodes**: different instances in the game (in the book H)
  - Initial node: no move yet (in the book $\varnothing$; in slides h_0)
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  - Active players: Bob at h_0 and Ann at h_1 (in the book $P(h_0) = \text{Bob}$ and $P(h_1) = \text{Ann}$)
- **Actions**: active player i has moves $A_i(h)$ at non-terminal node $h \in H \setminus Z$
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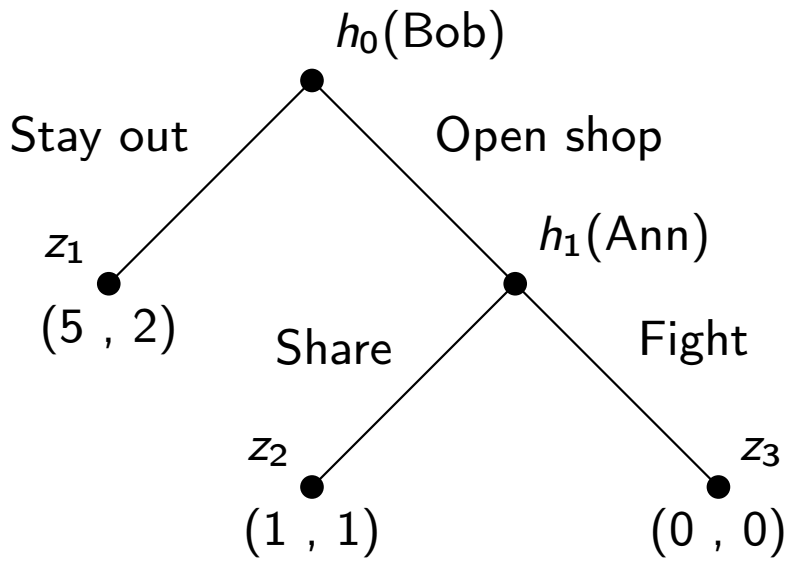

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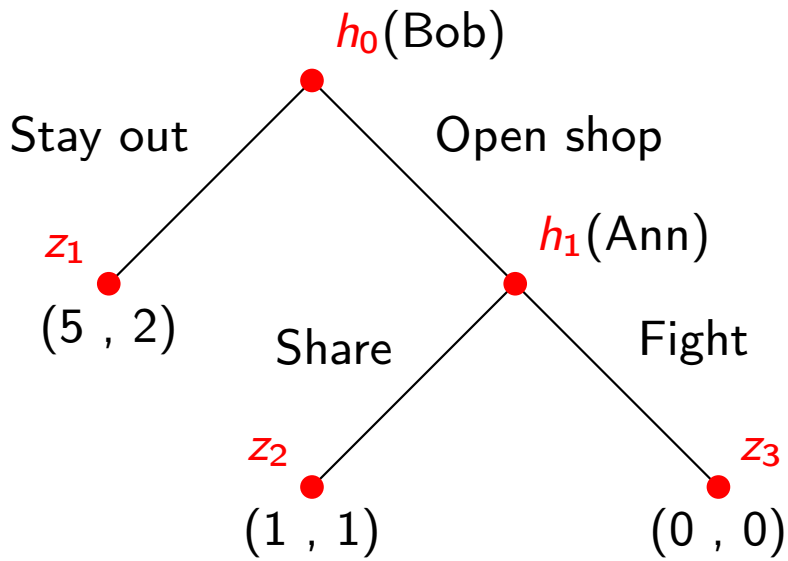

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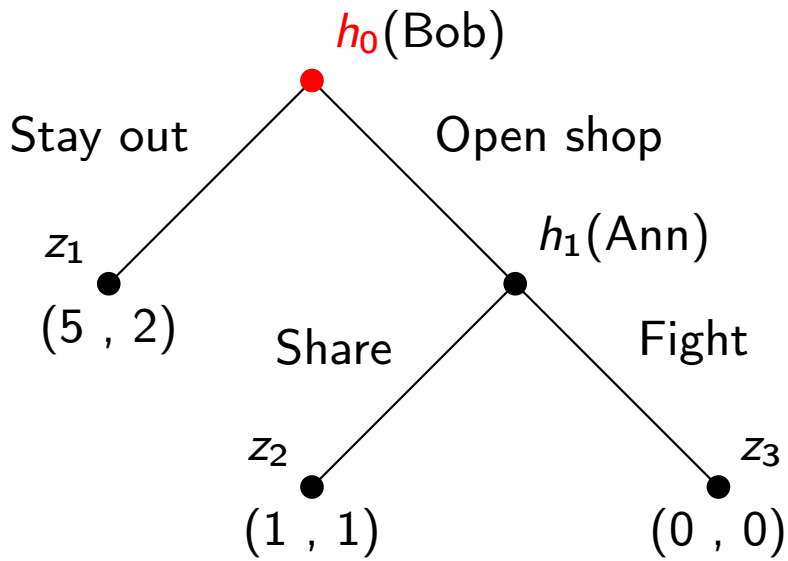

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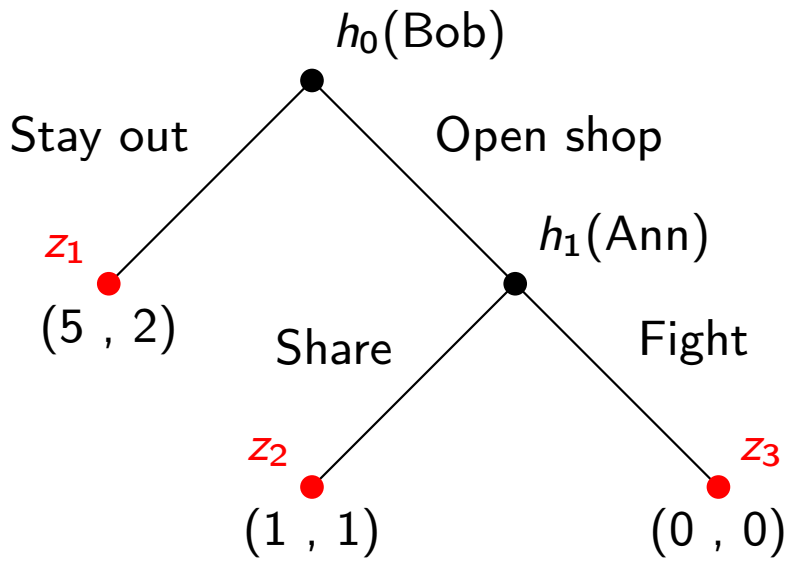

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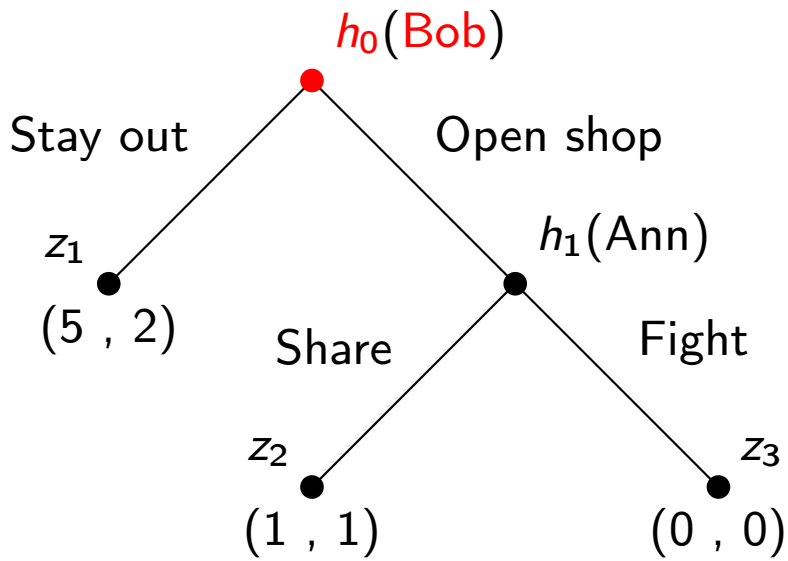

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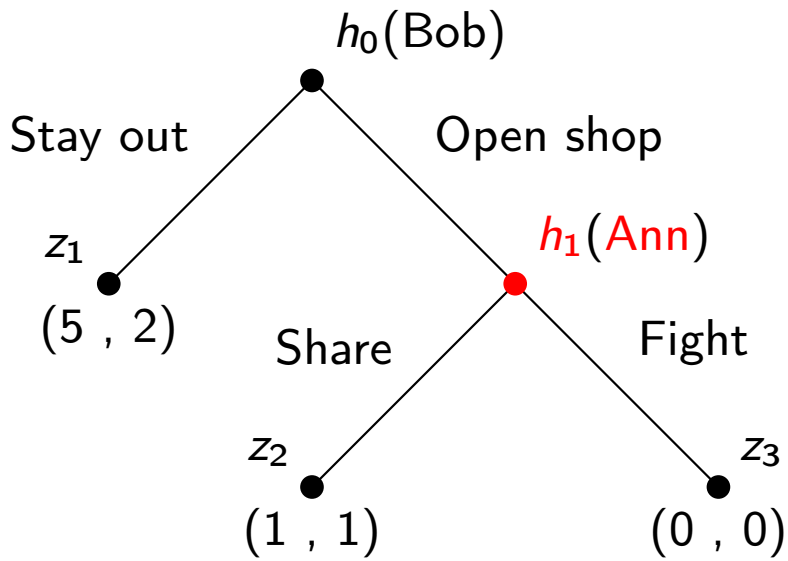

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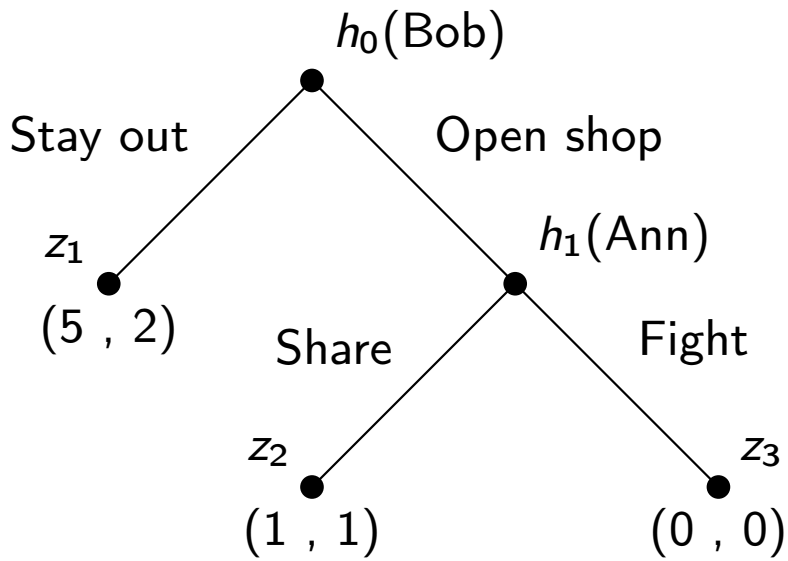

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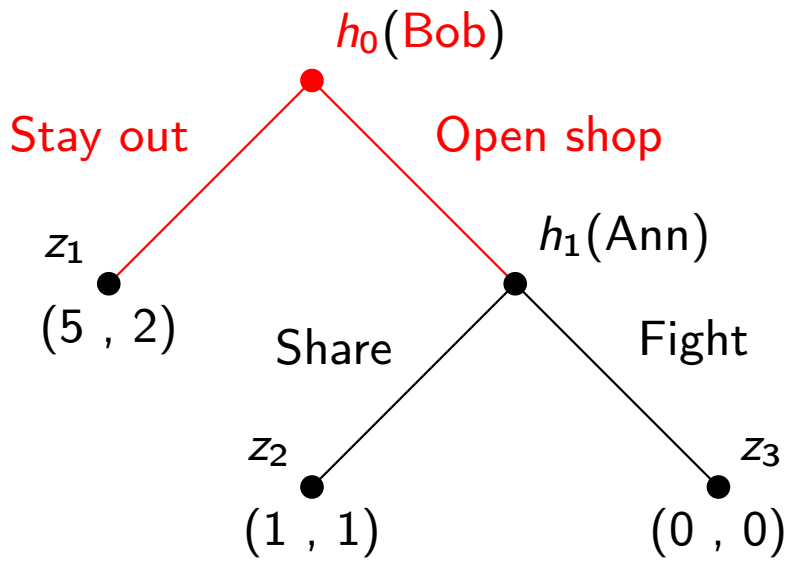

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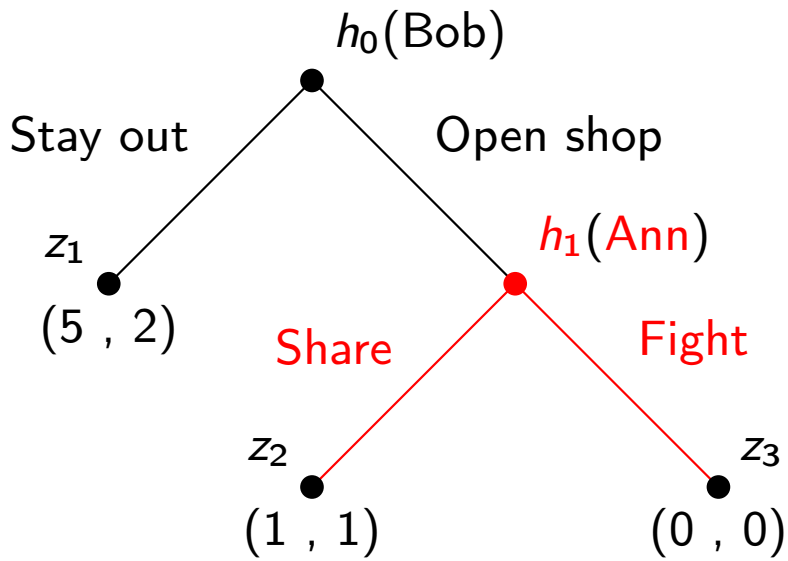
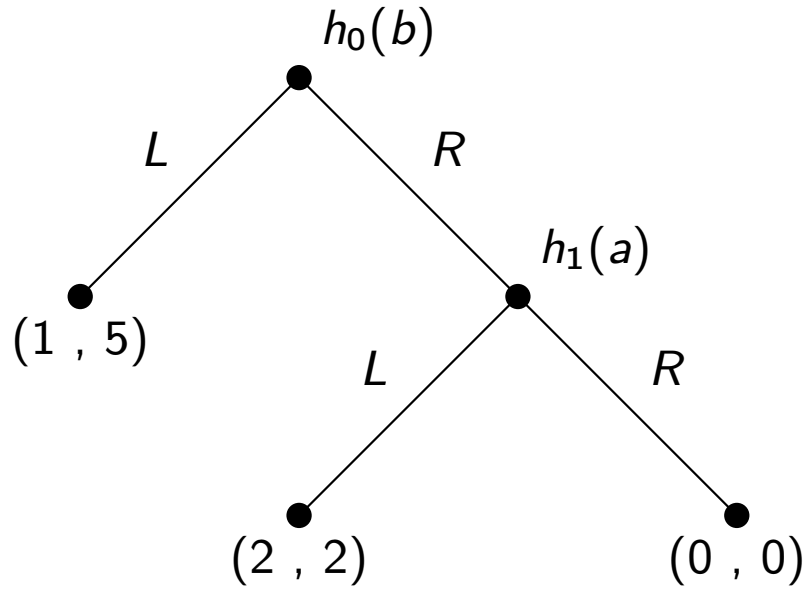


- We will now explain what a strategy is (key concept)
- In extensive games actions and strategies are different, when the player moves more than once
- How do you understand strategy intuitively?

- **Strategy**: contingent plan (one action per non-terminal node)
- Not all nodes realized, but the player is always prepared to act

Example (One action only)



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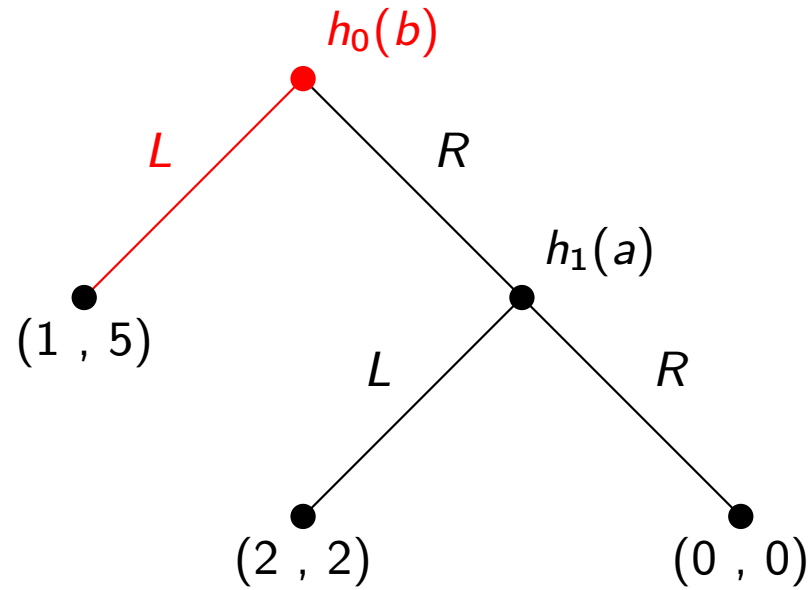

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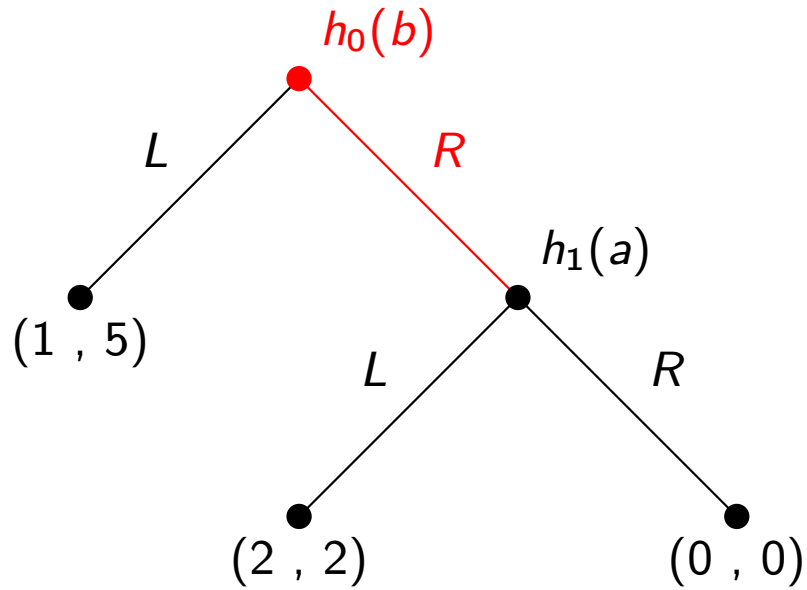

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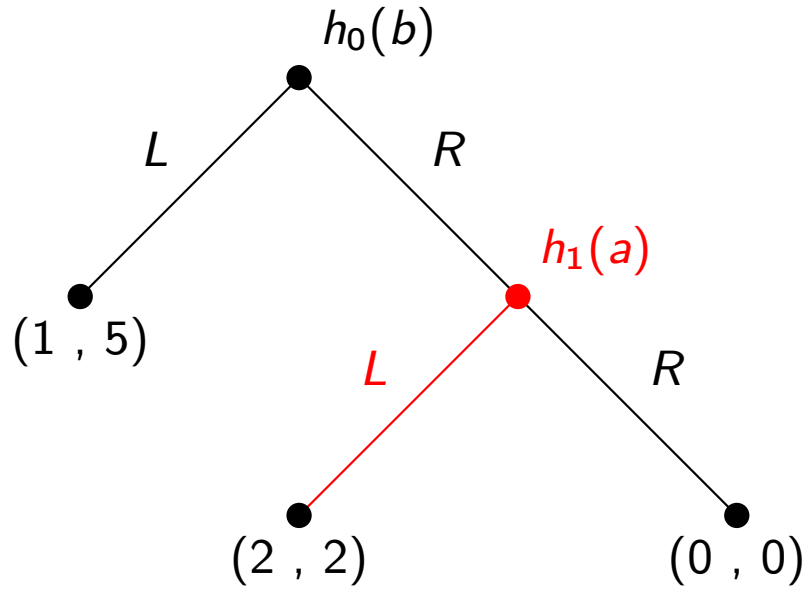

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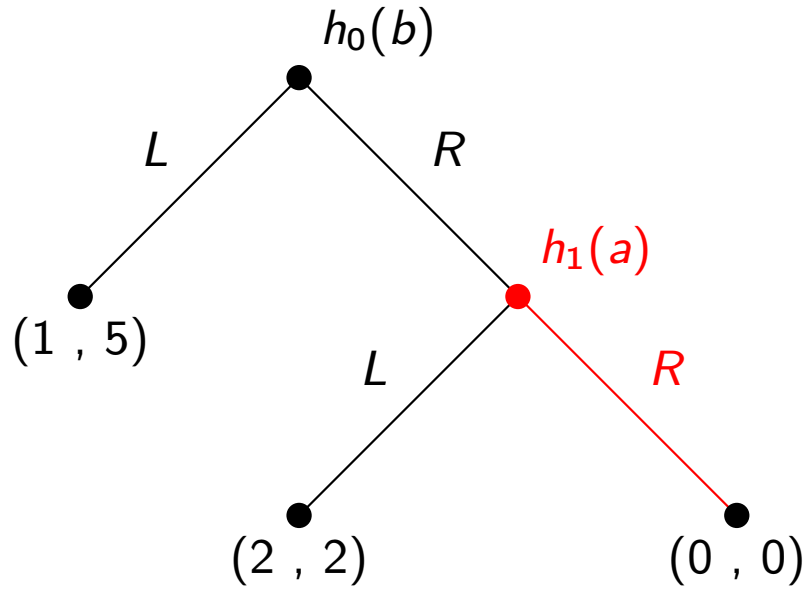

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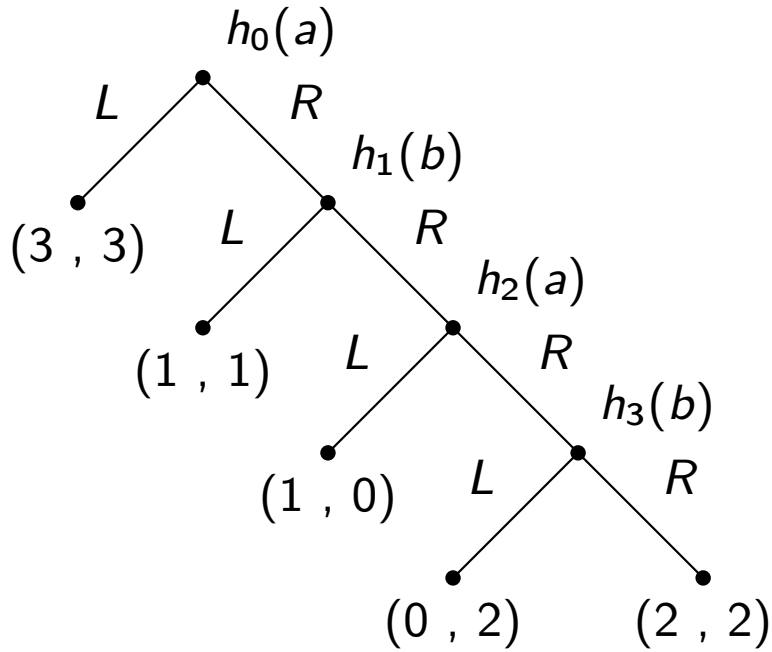

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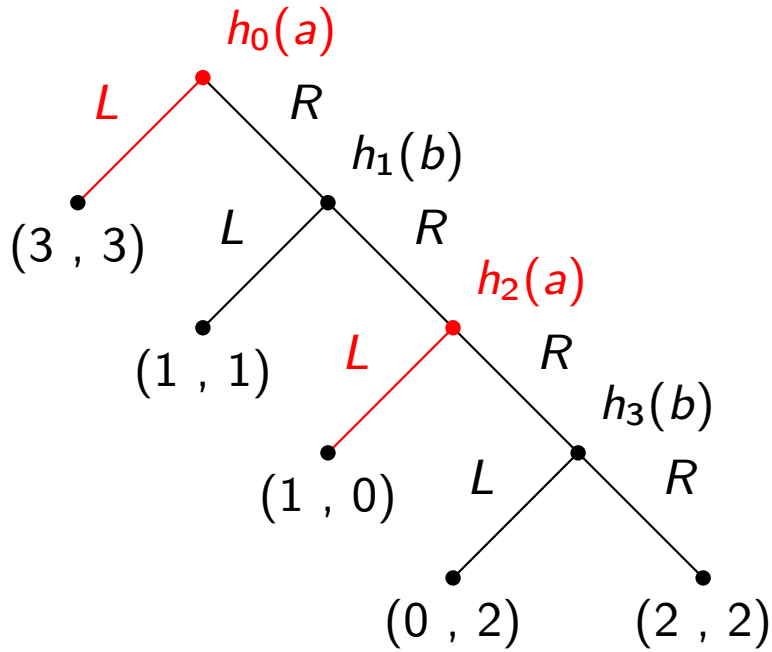

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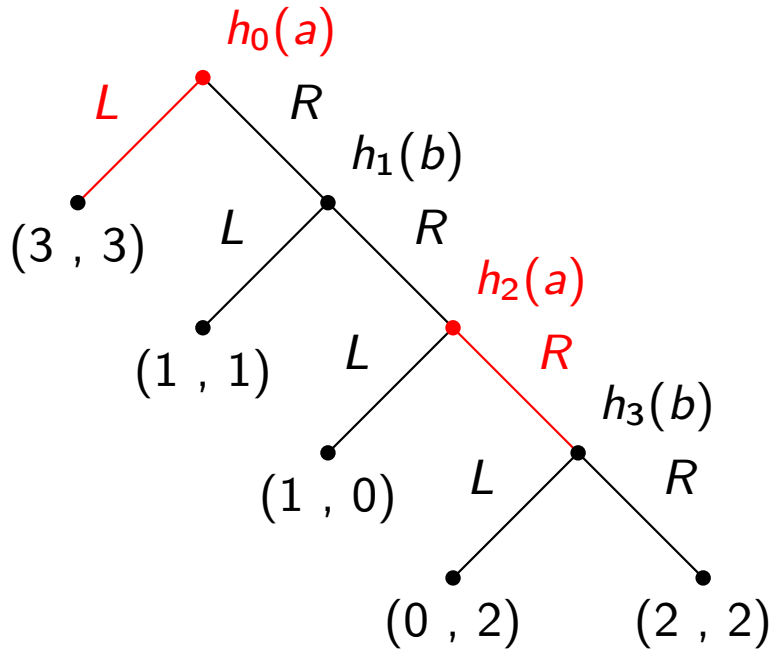

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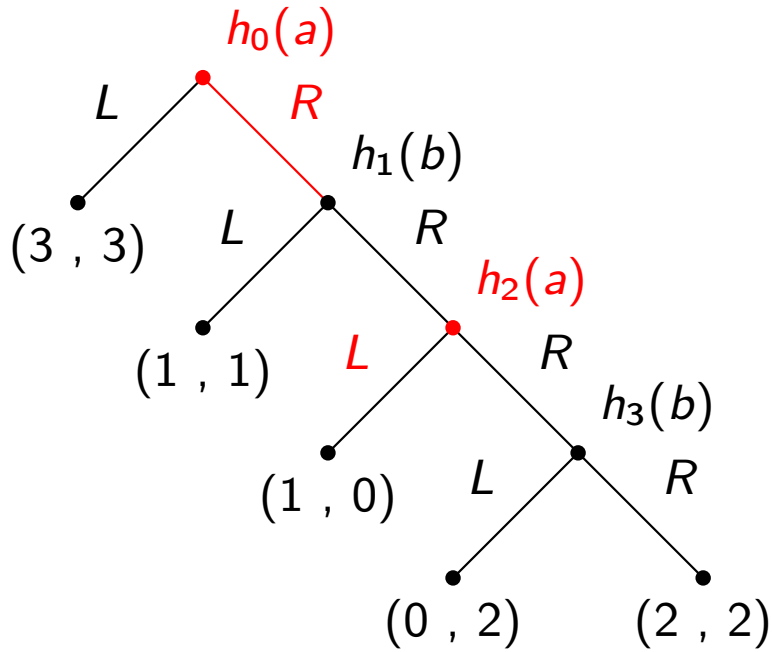

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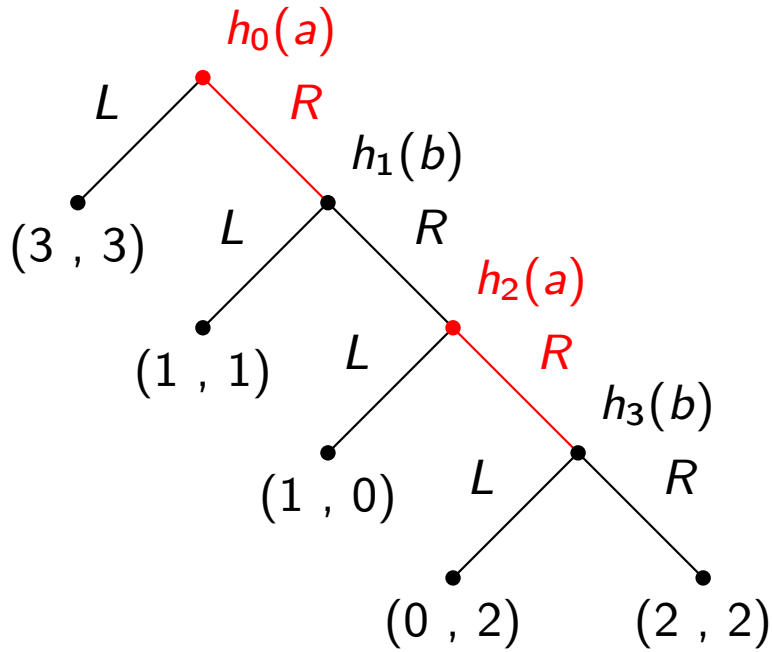

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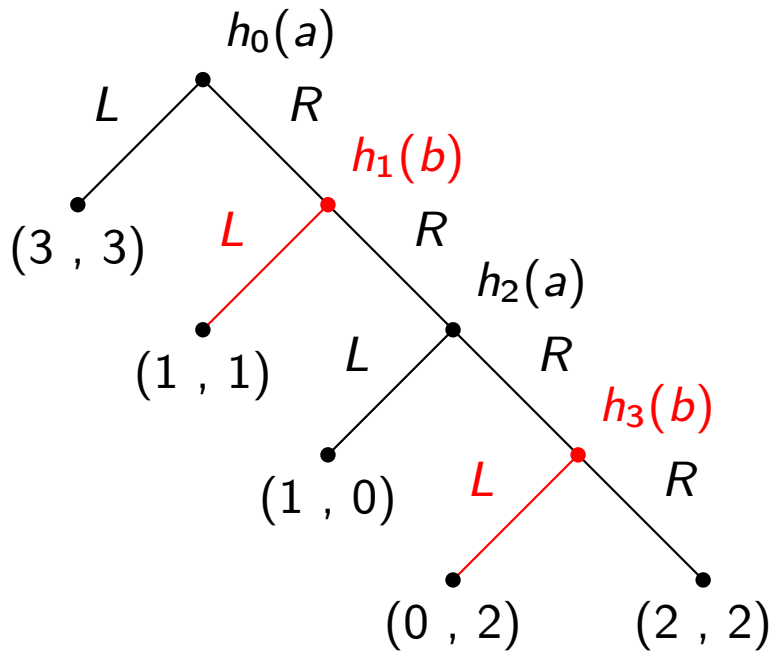

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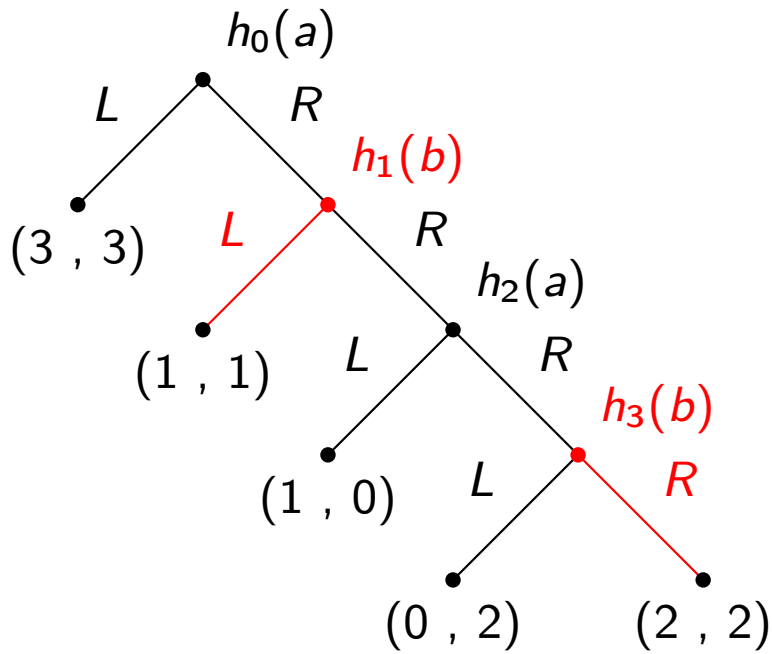

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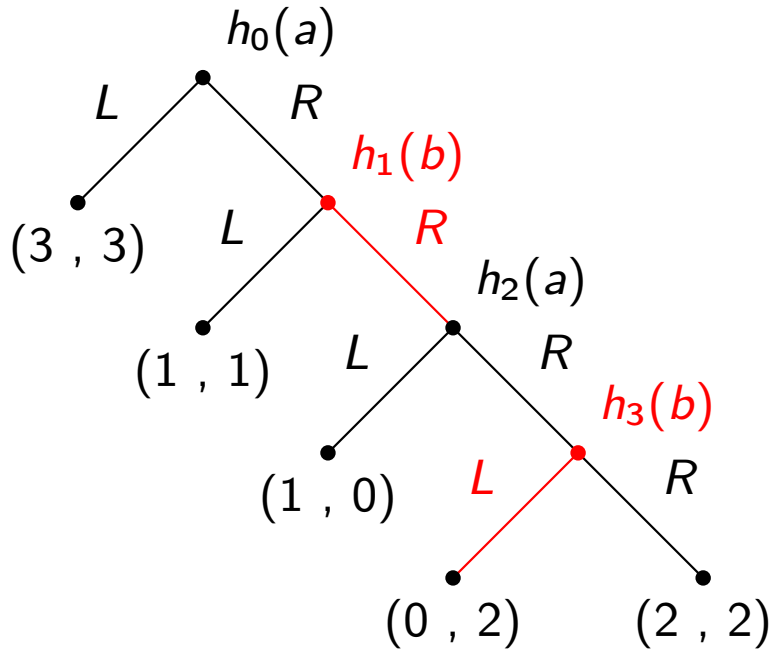

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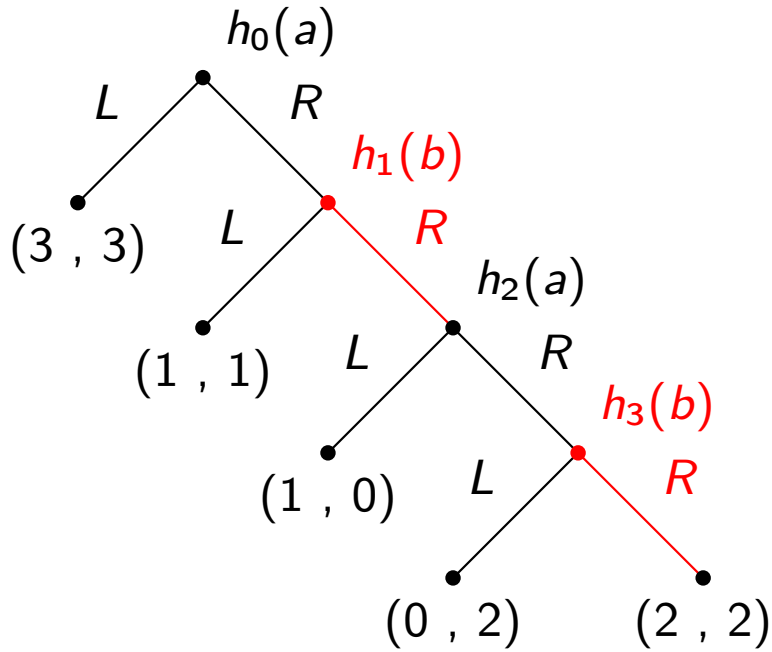

$$S^a = \{LL, LR, RL, RR\} \text{ (first letter for } h_0\text{, second letter for } h_2\text{)}$$
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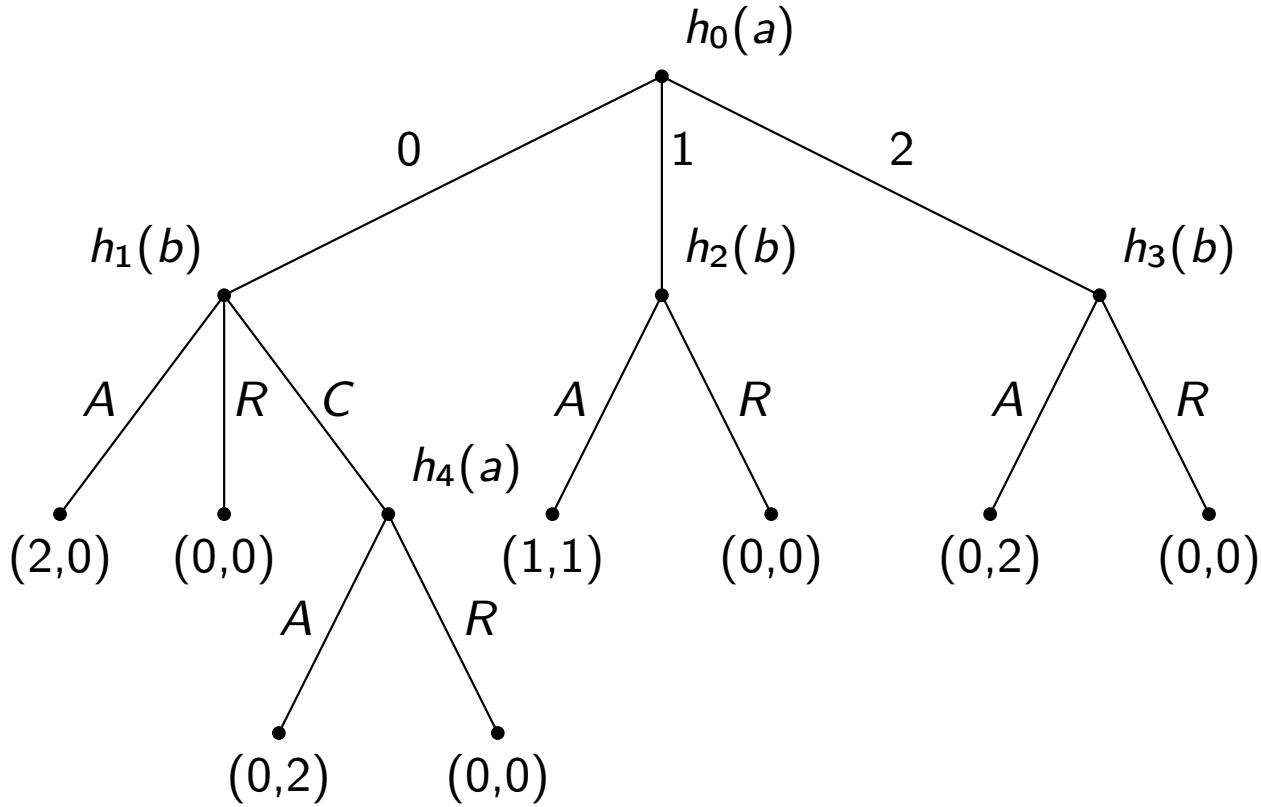
- Not all nodes realized, but the player is always prepared to act

Example (Mutually exclusive nodes)



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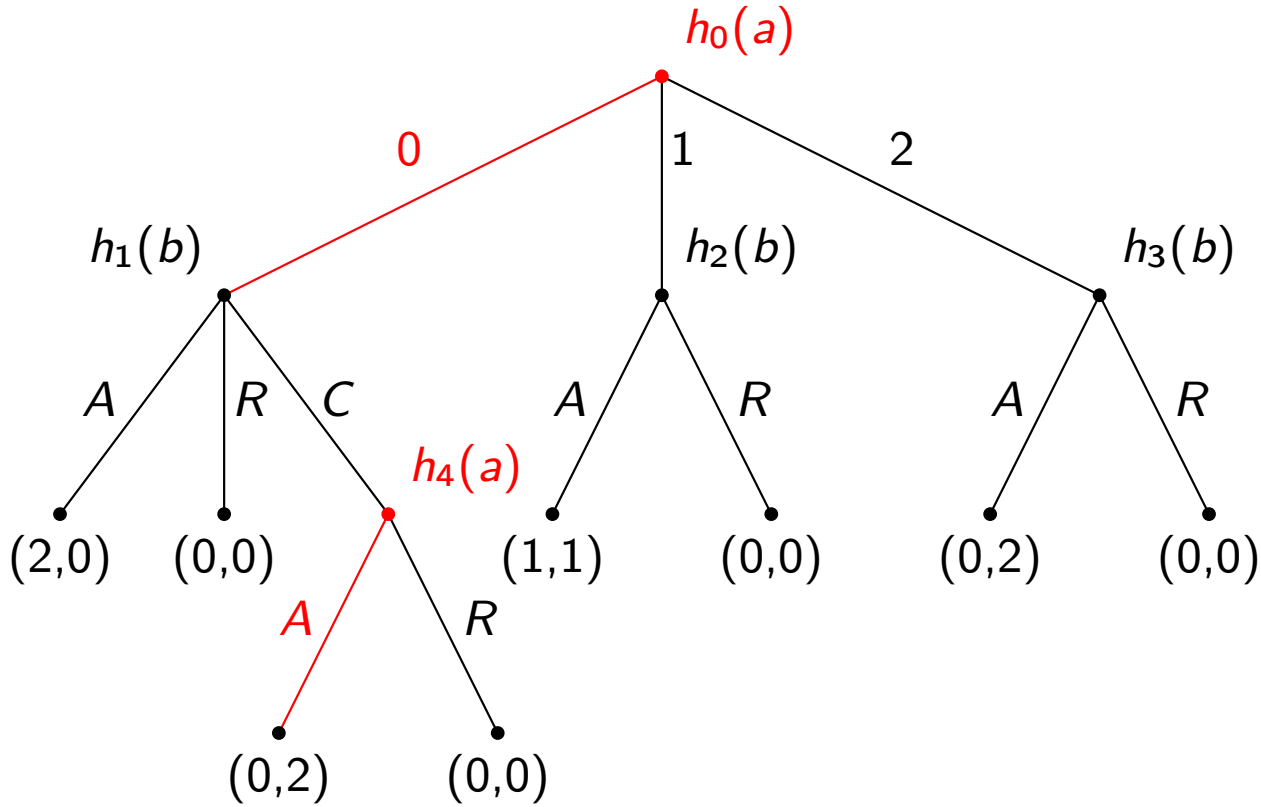

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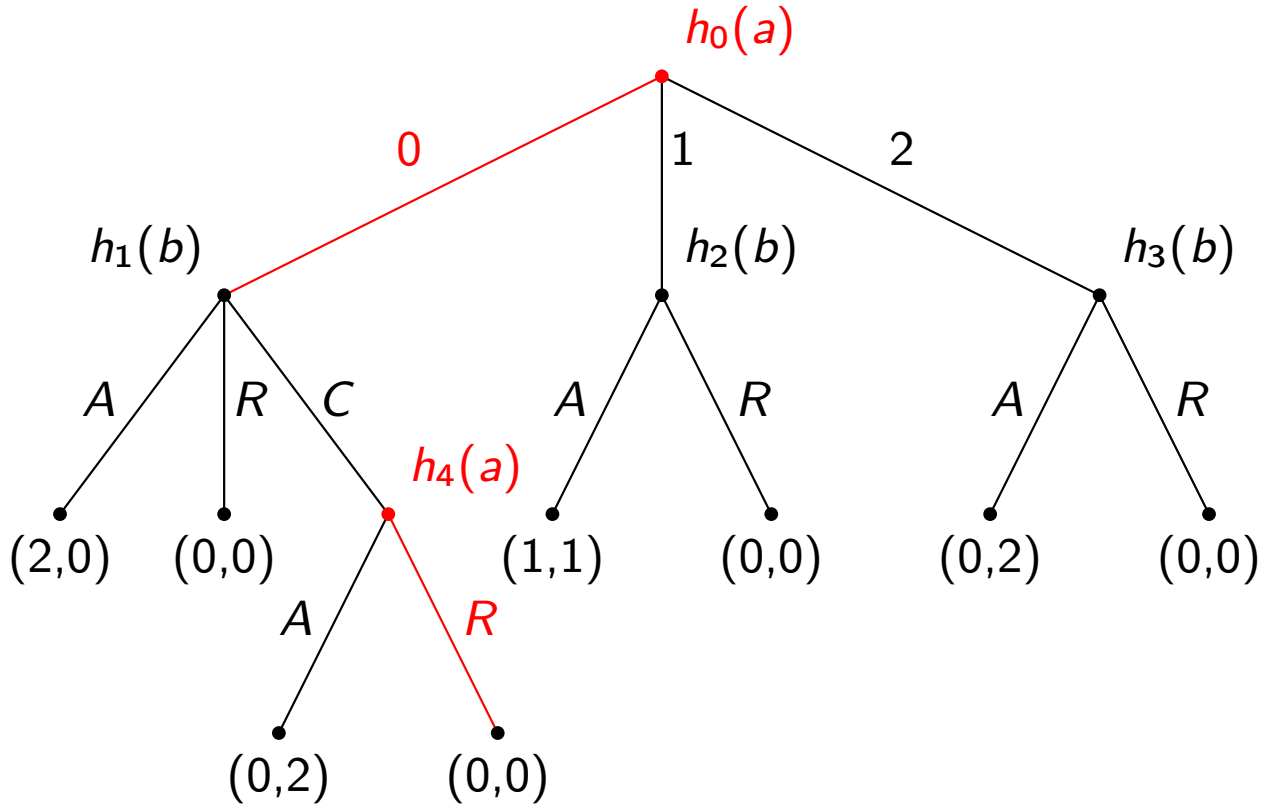

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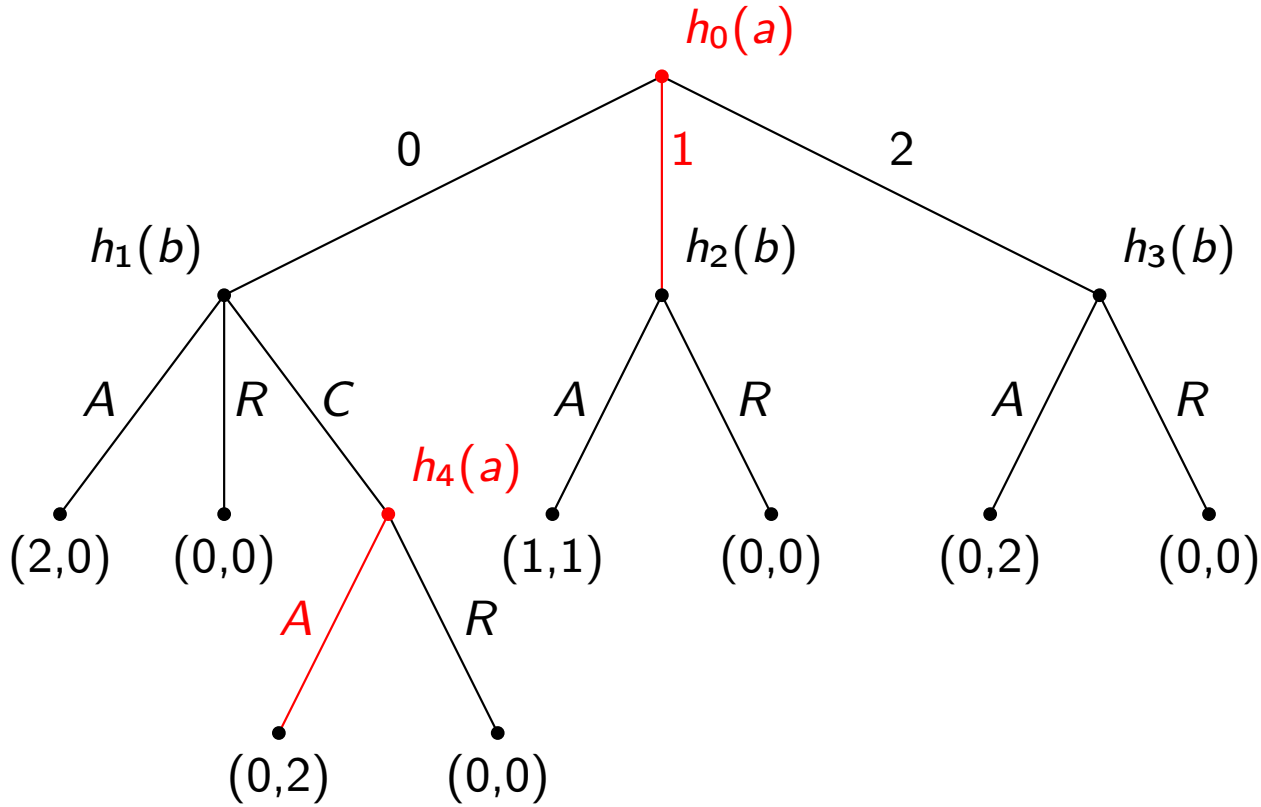

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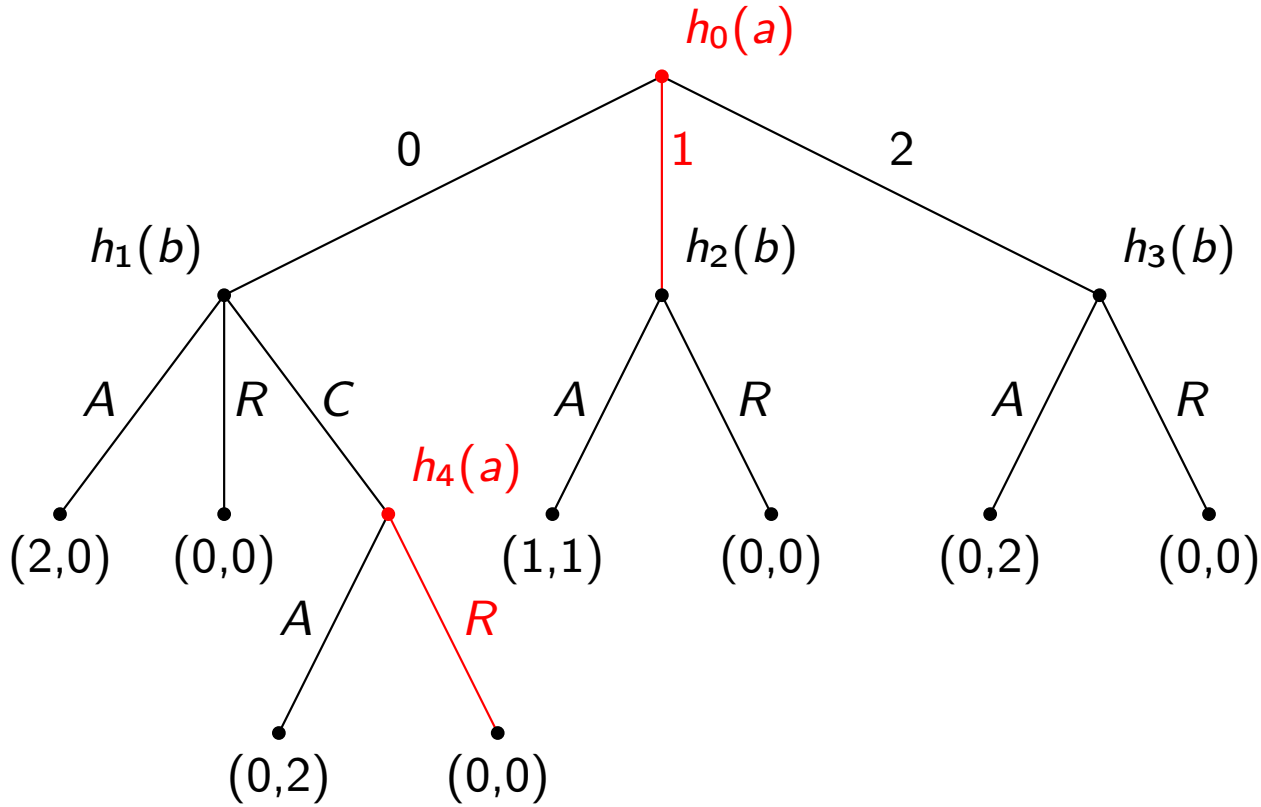

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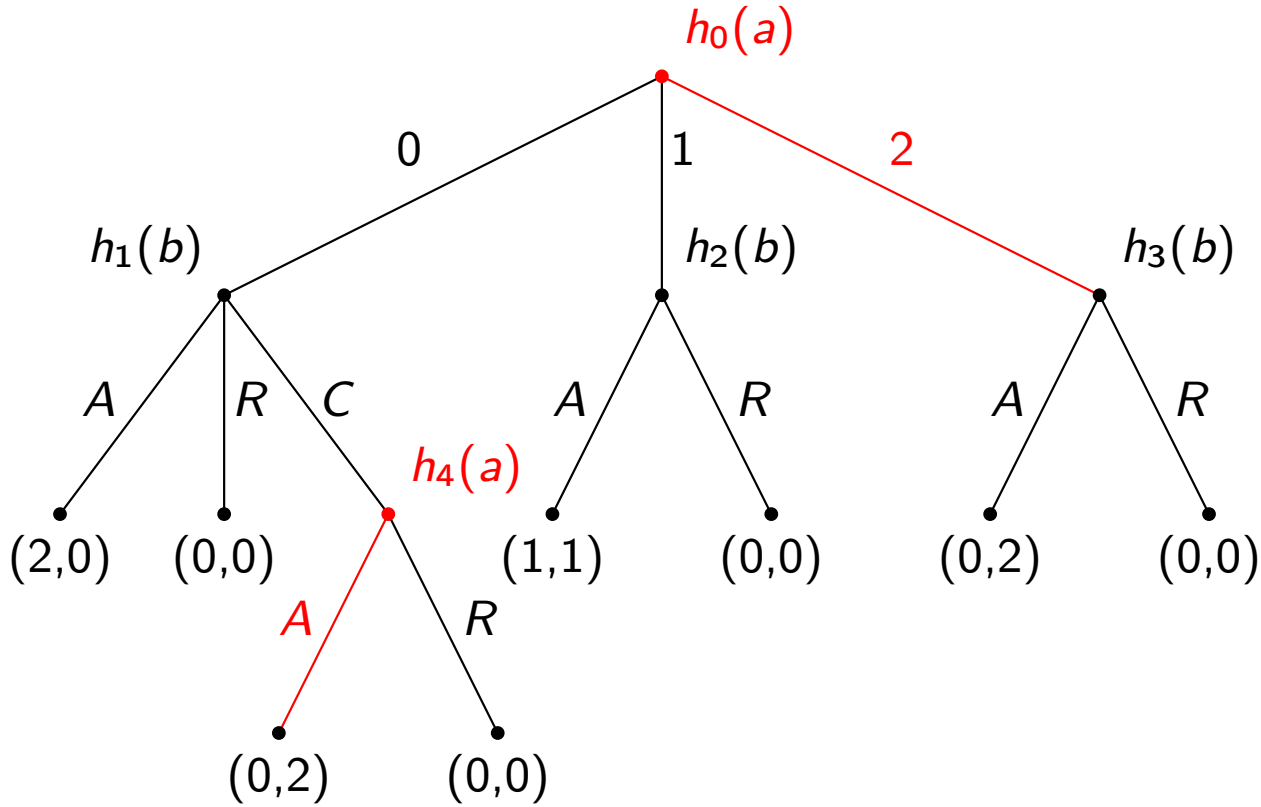

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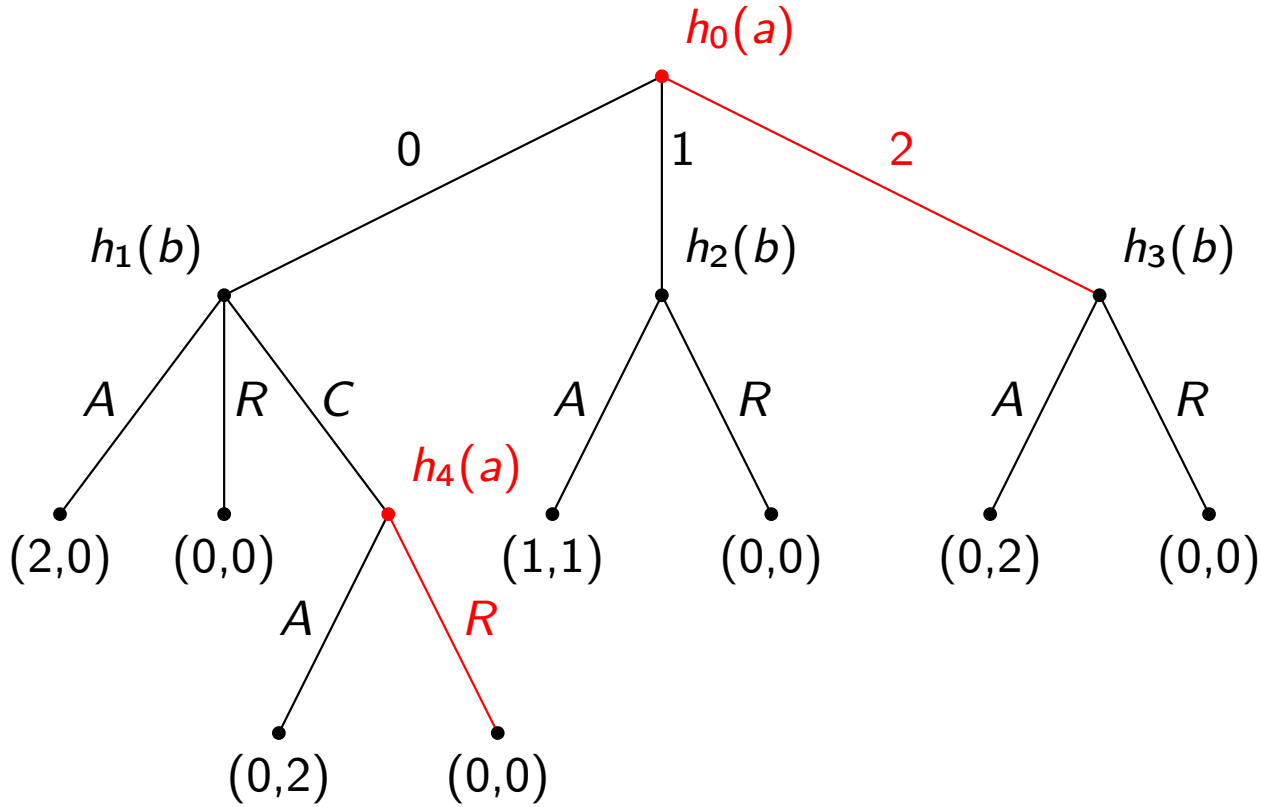

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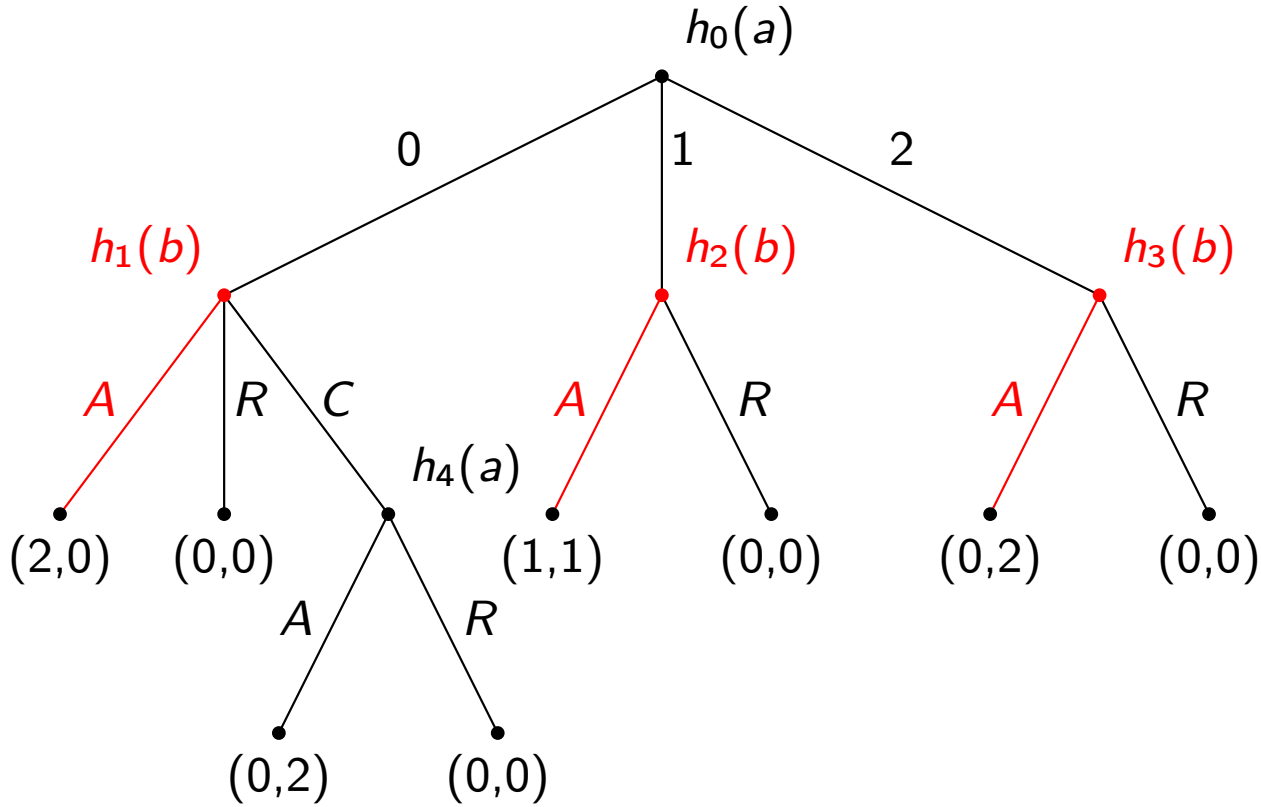

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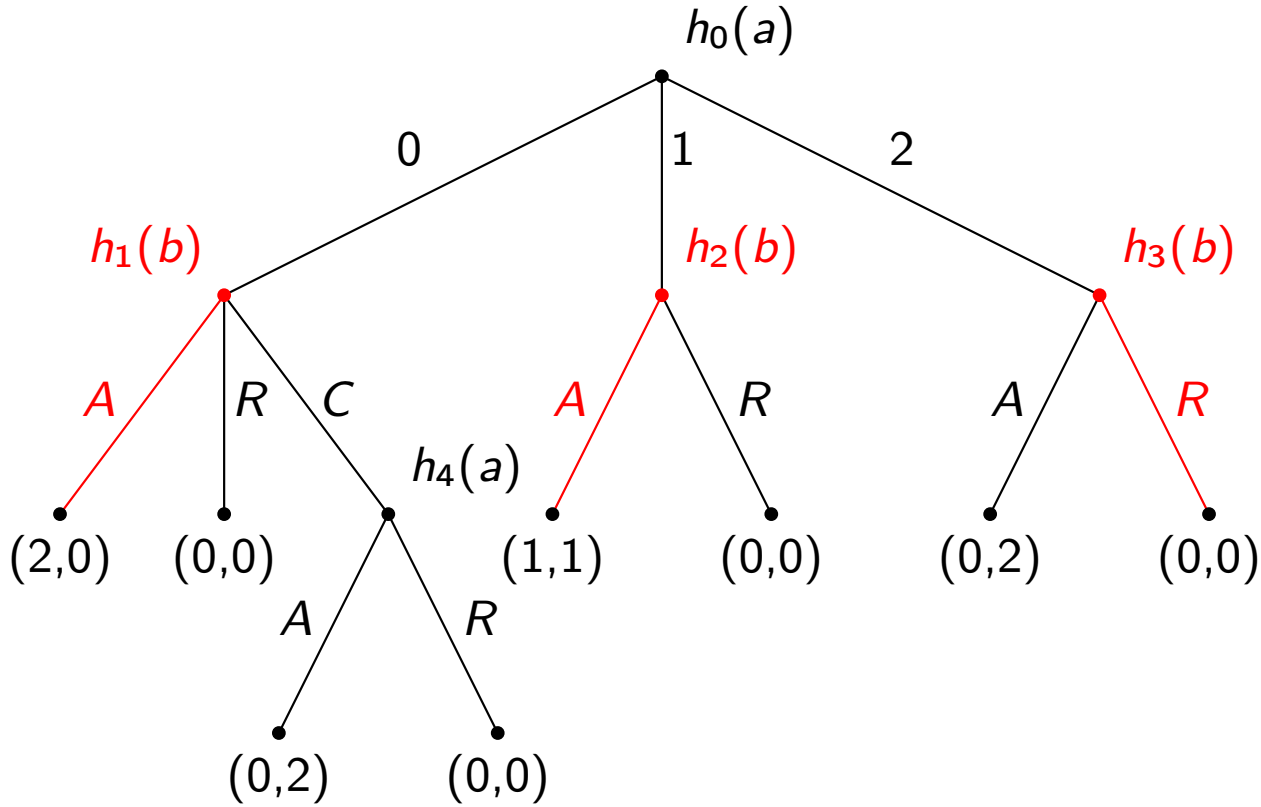

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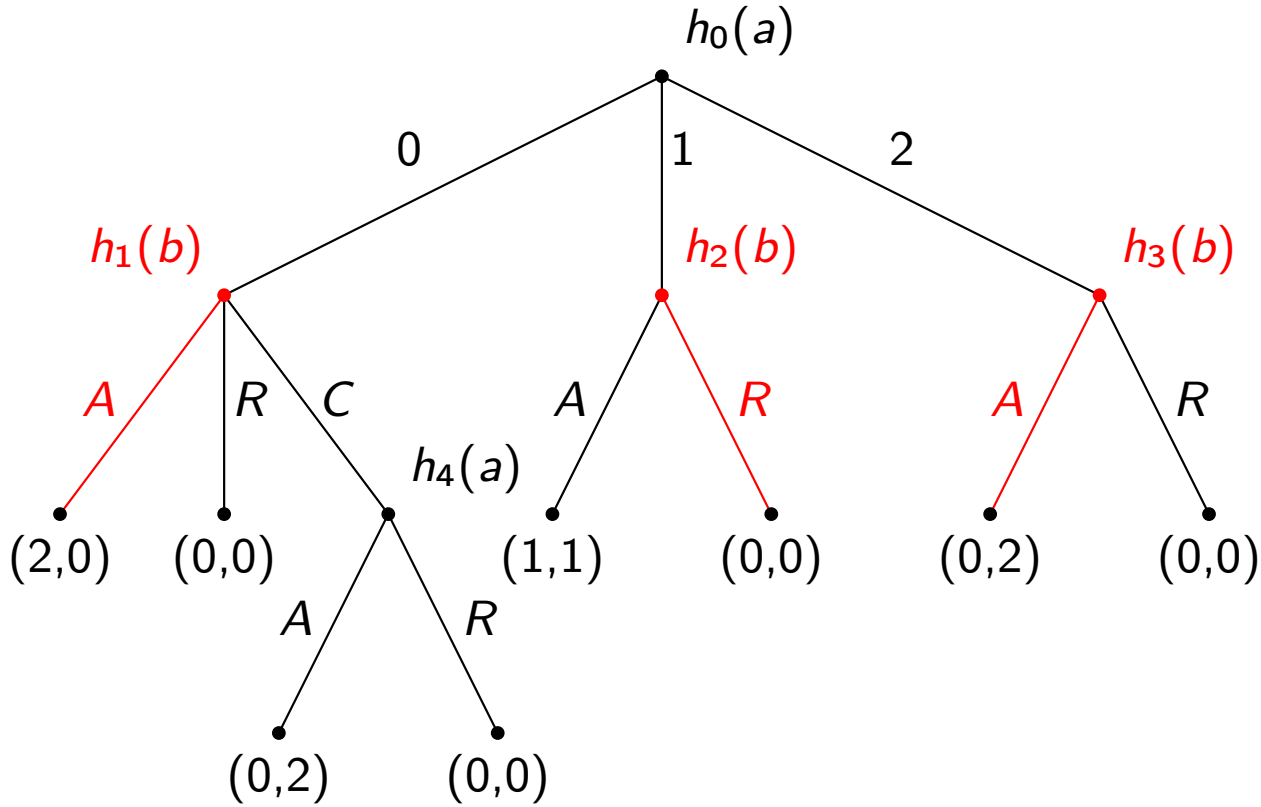

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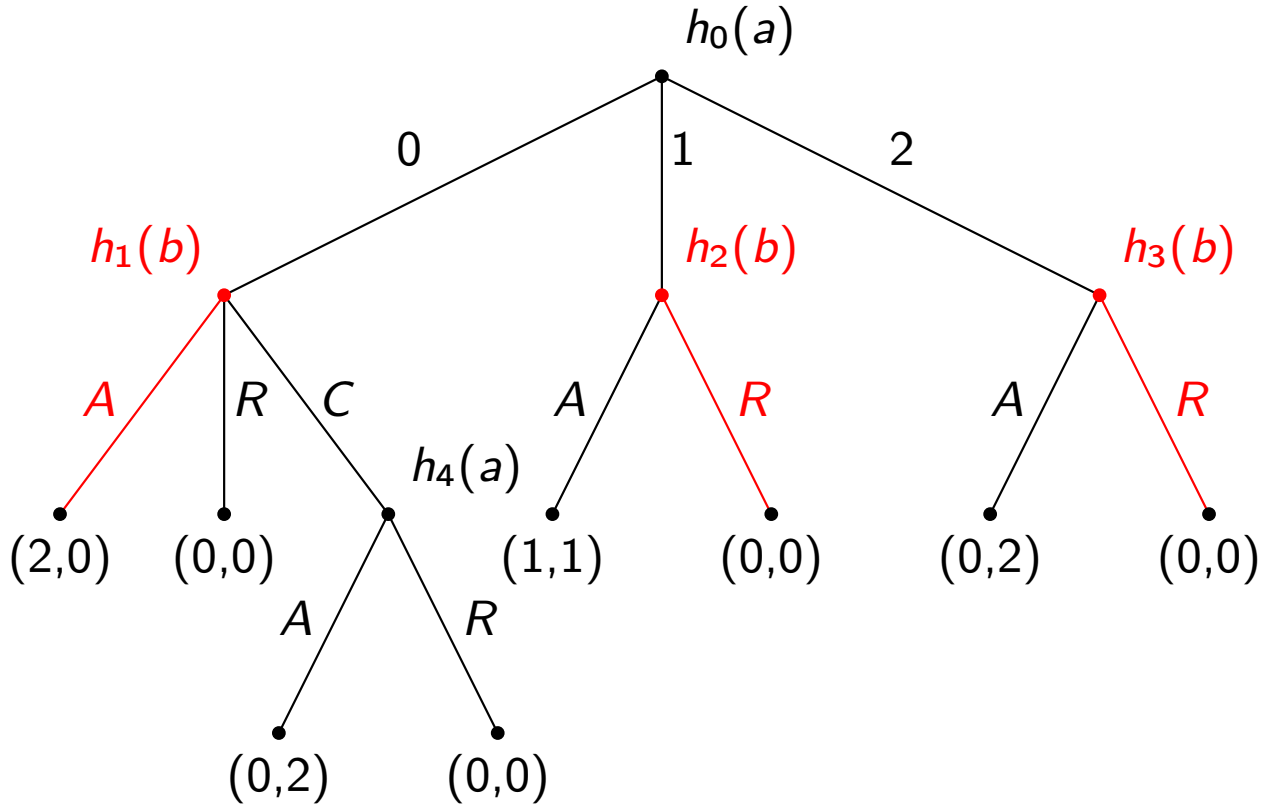

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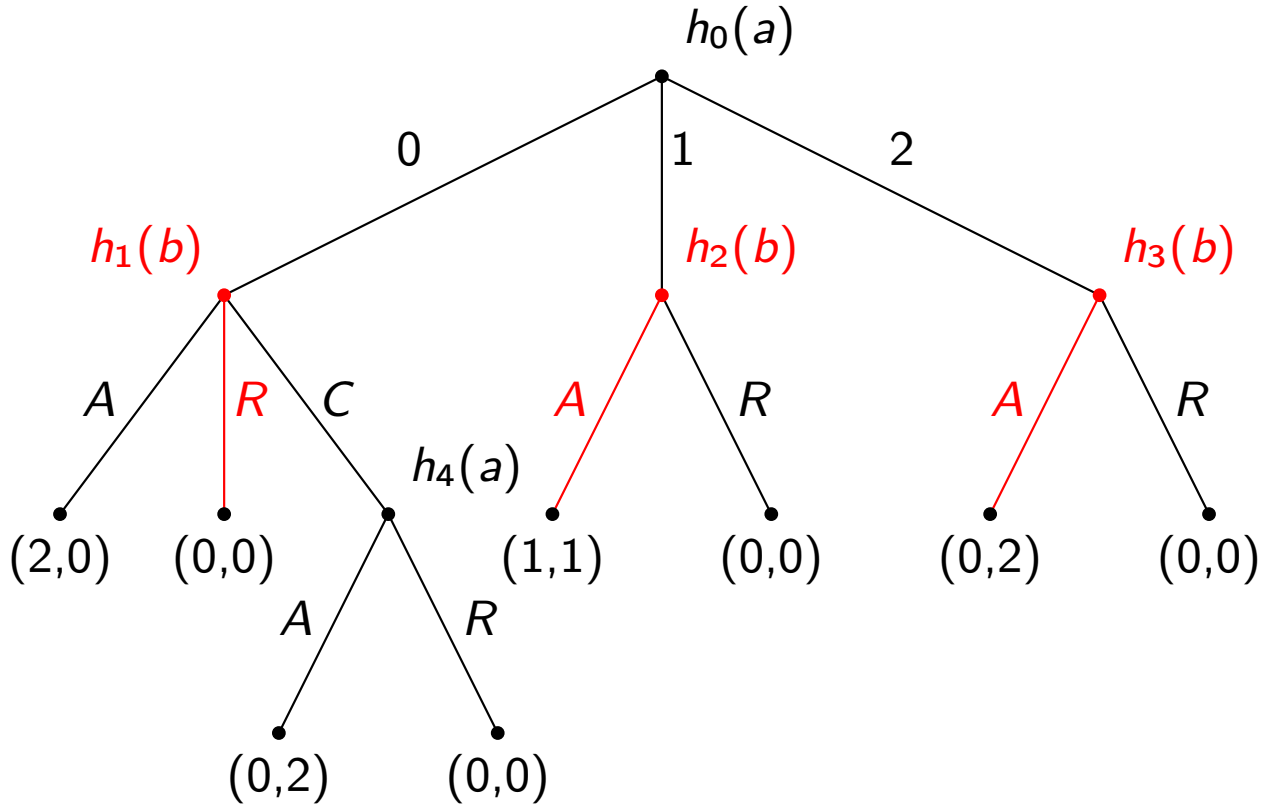

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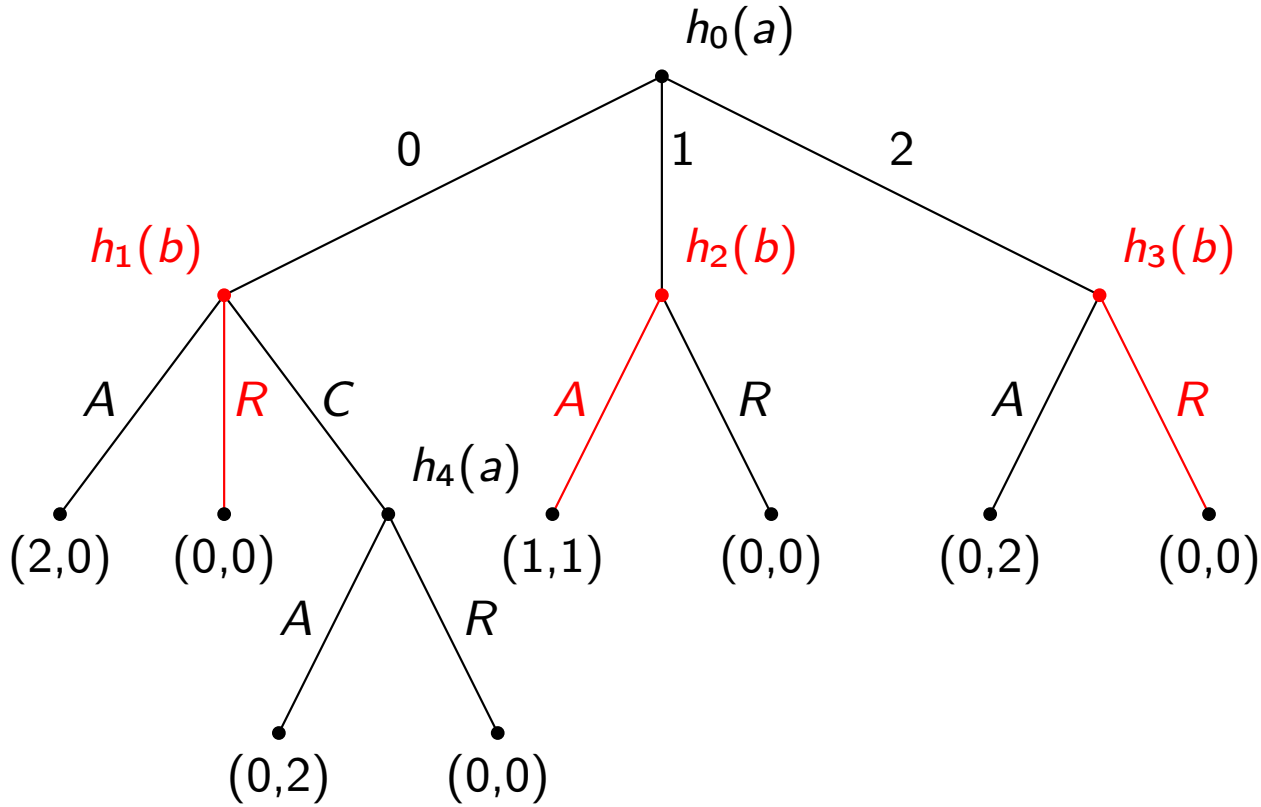

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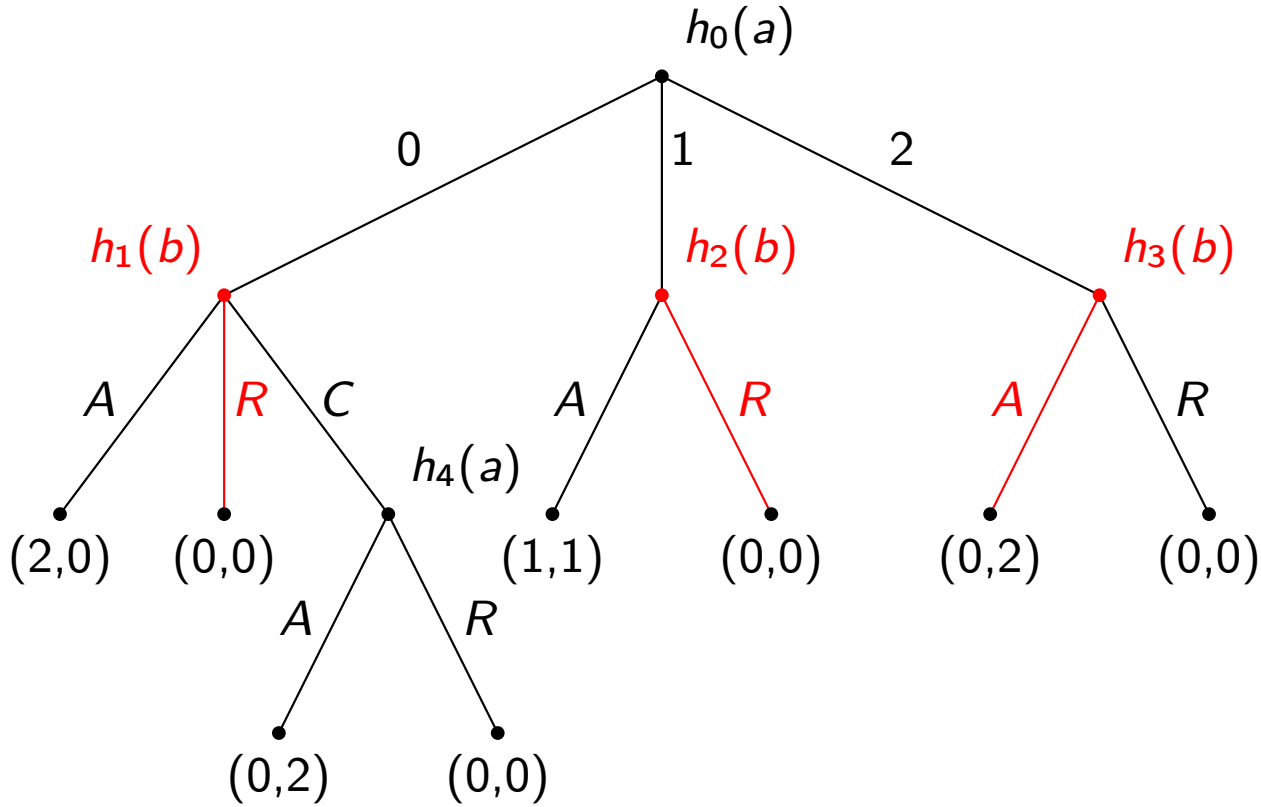

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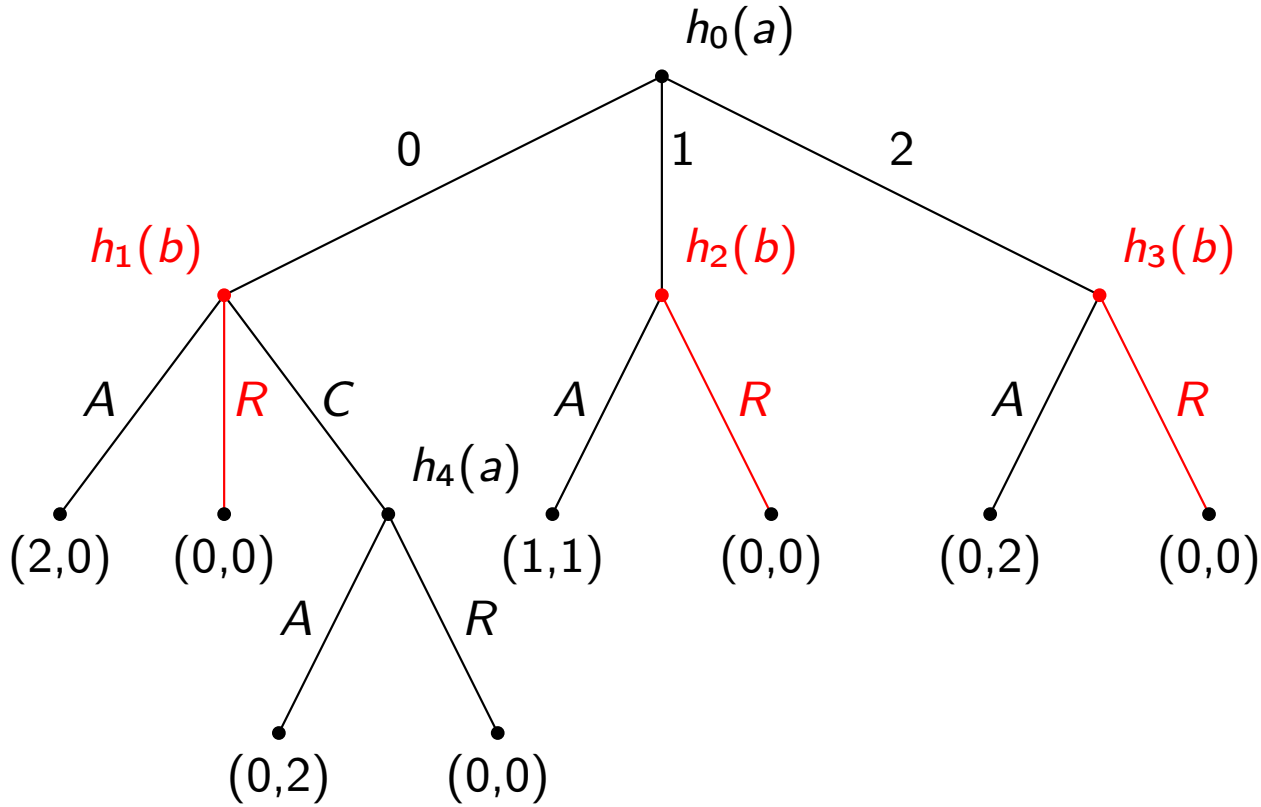

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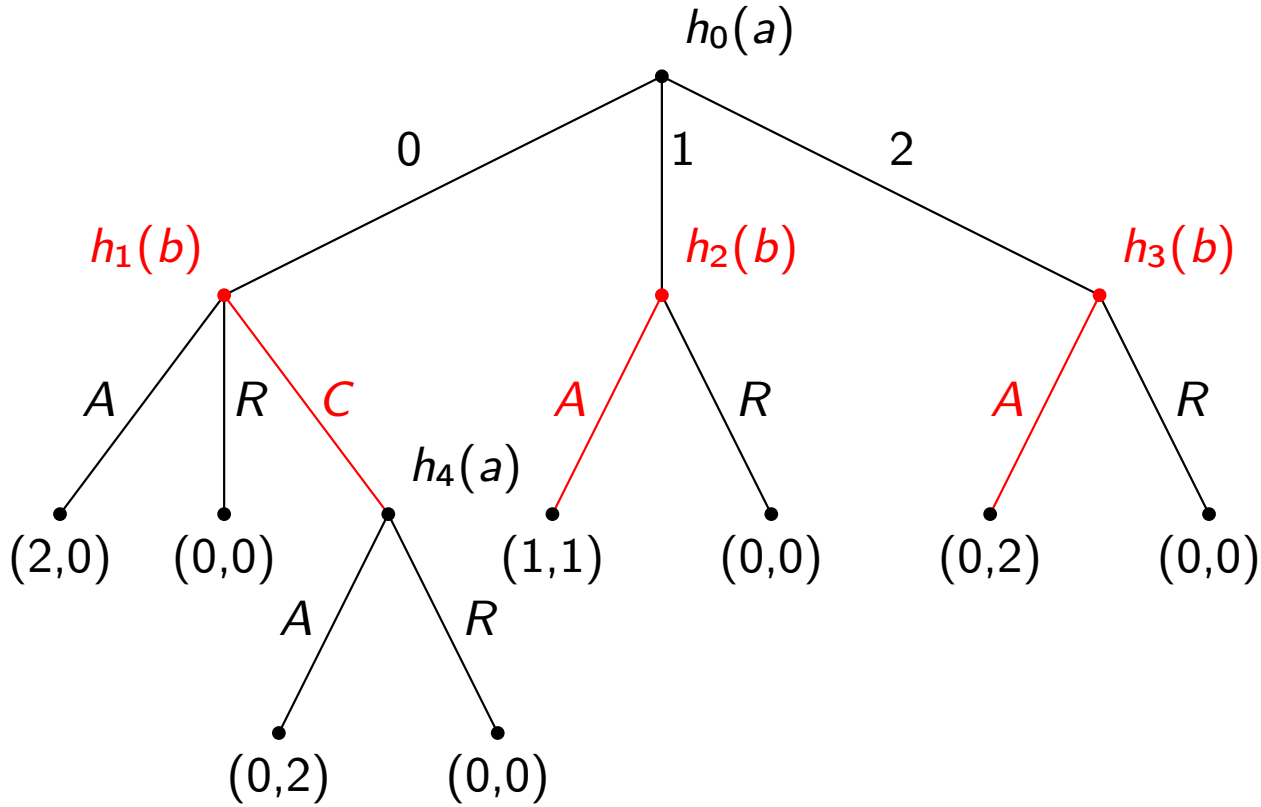

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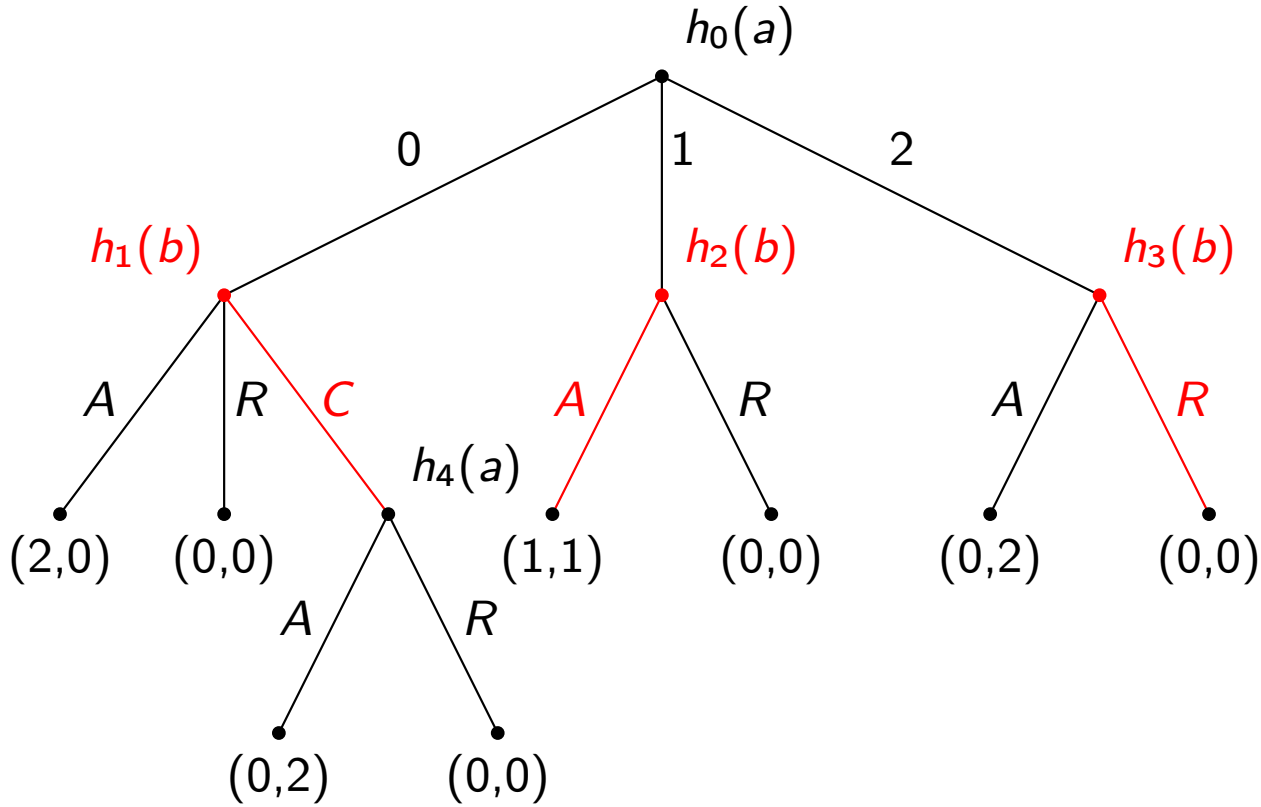

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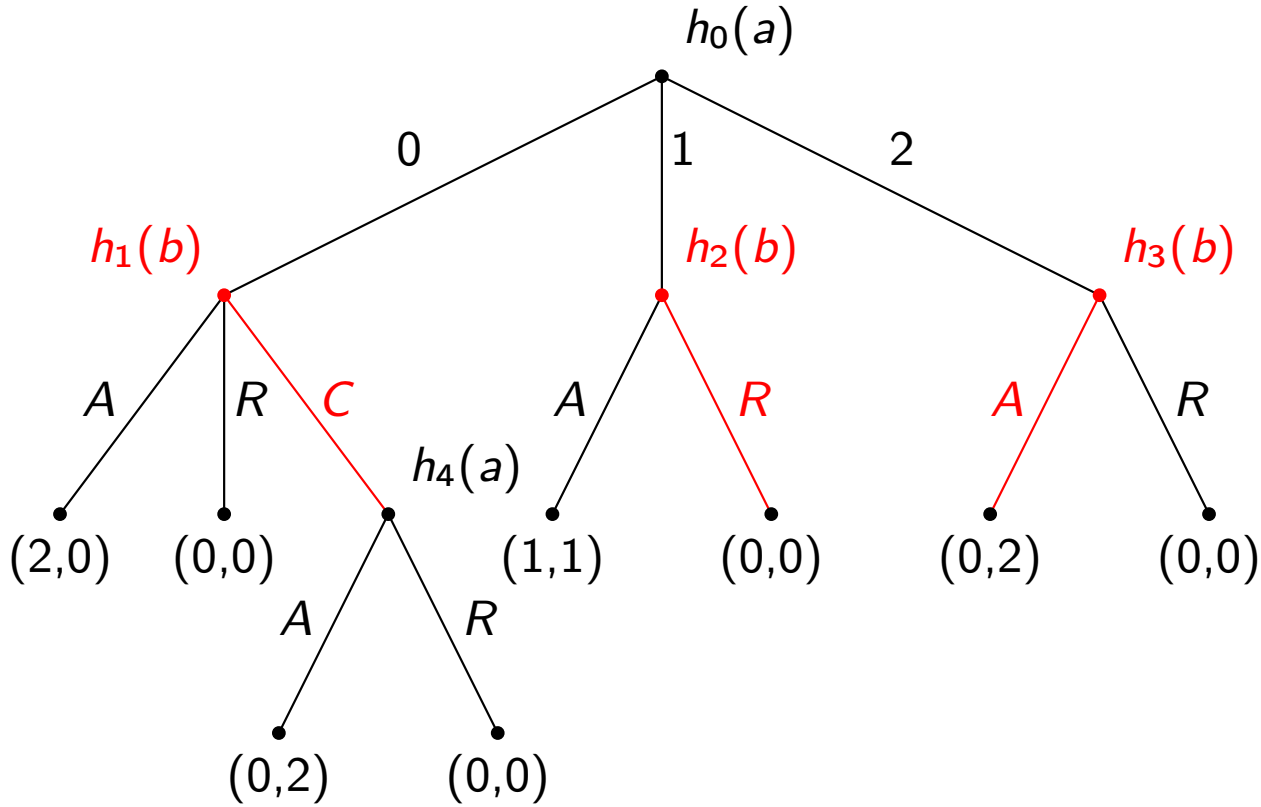

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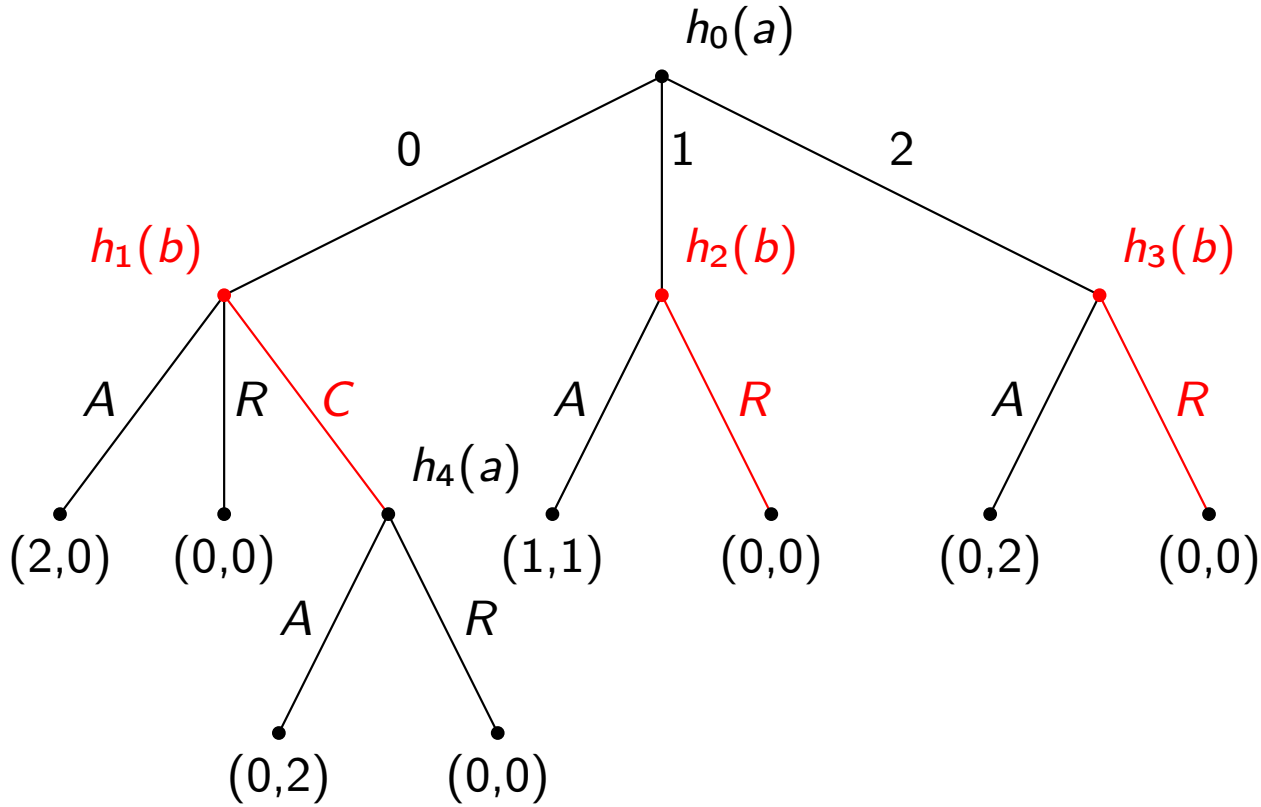

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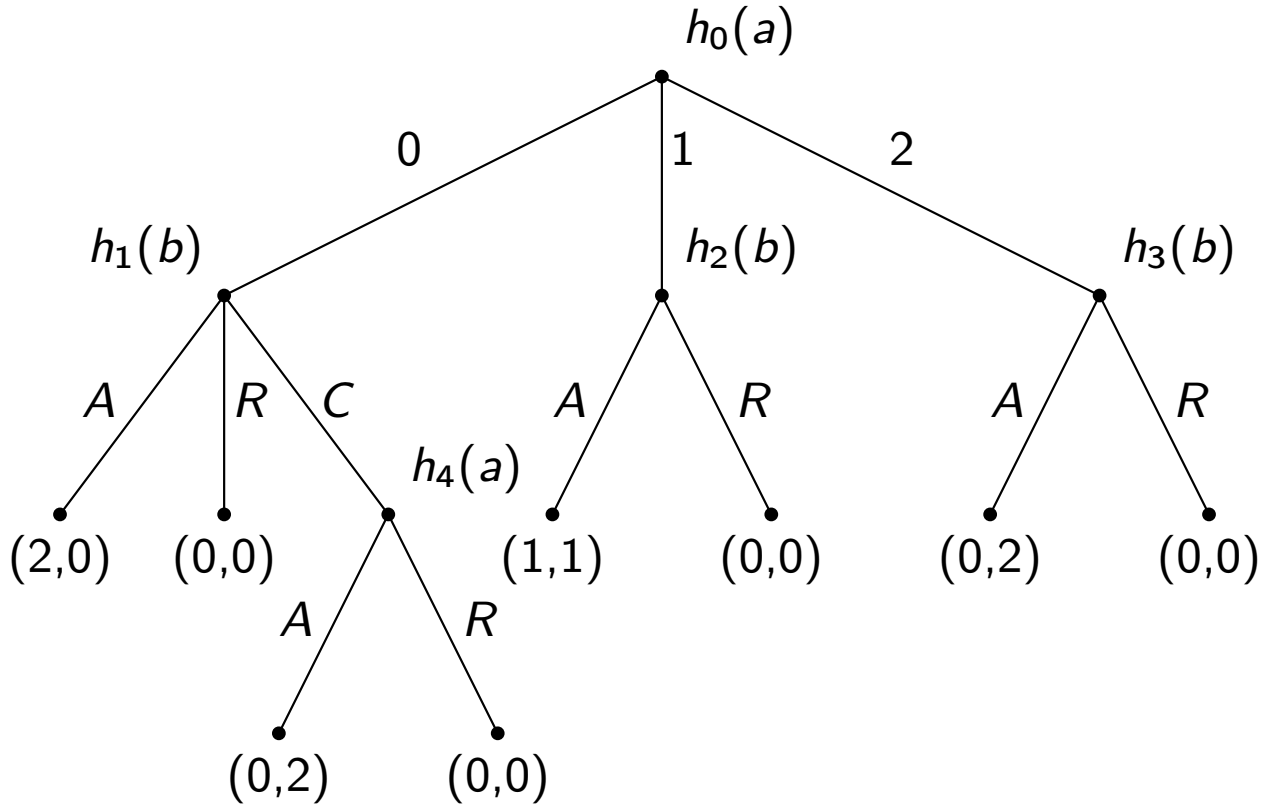

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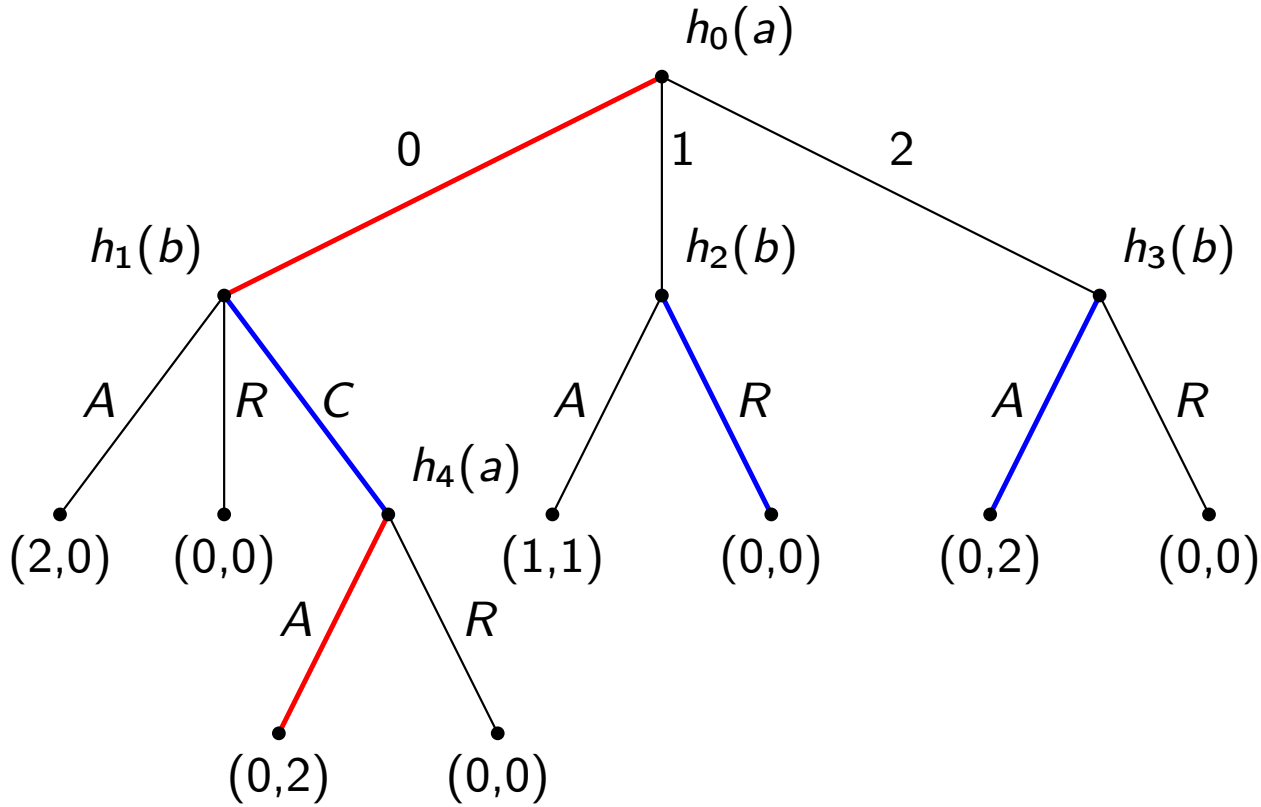

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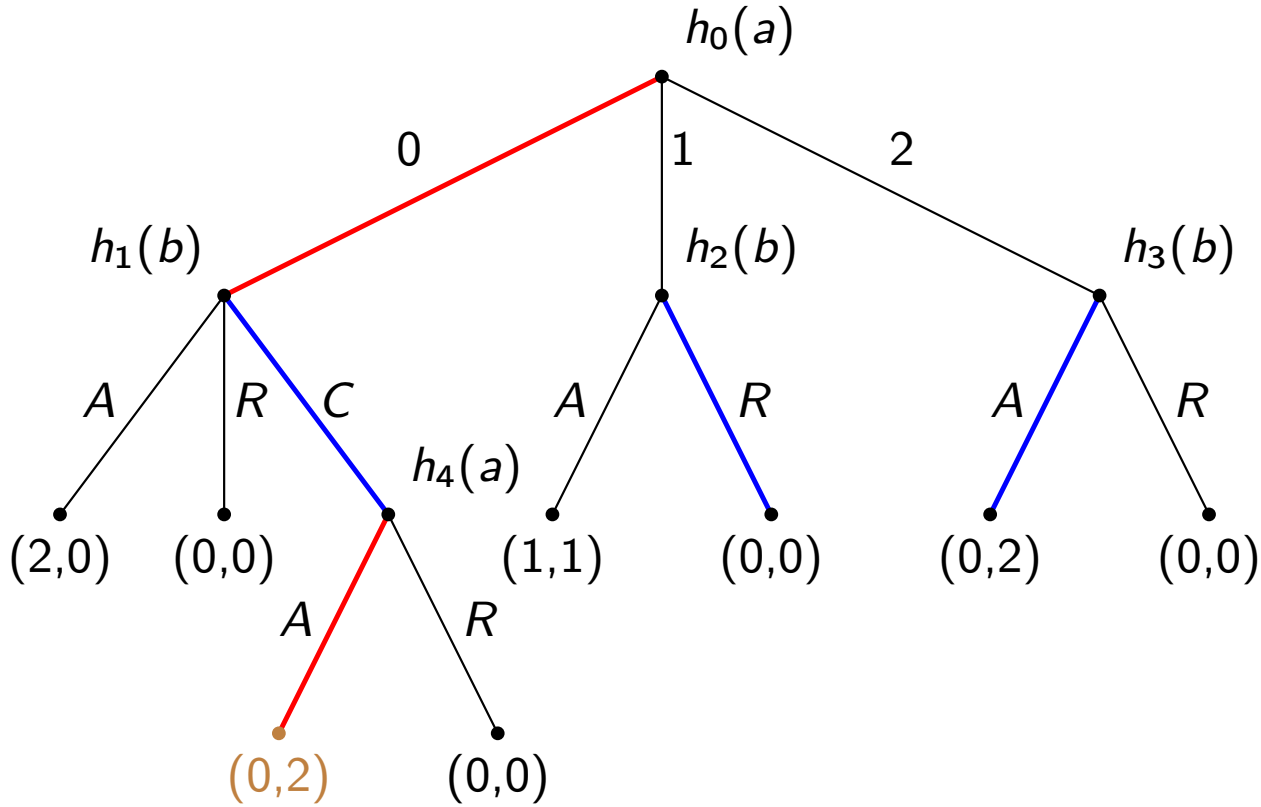

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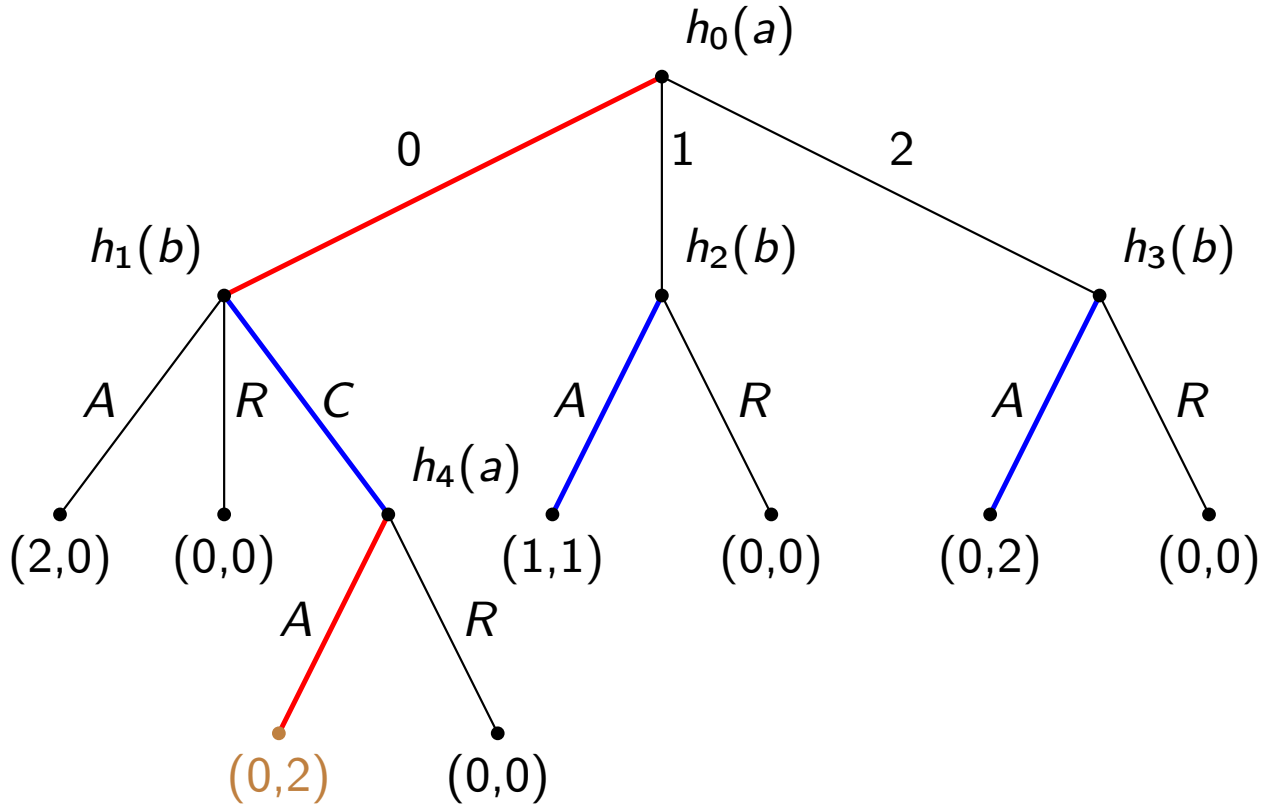

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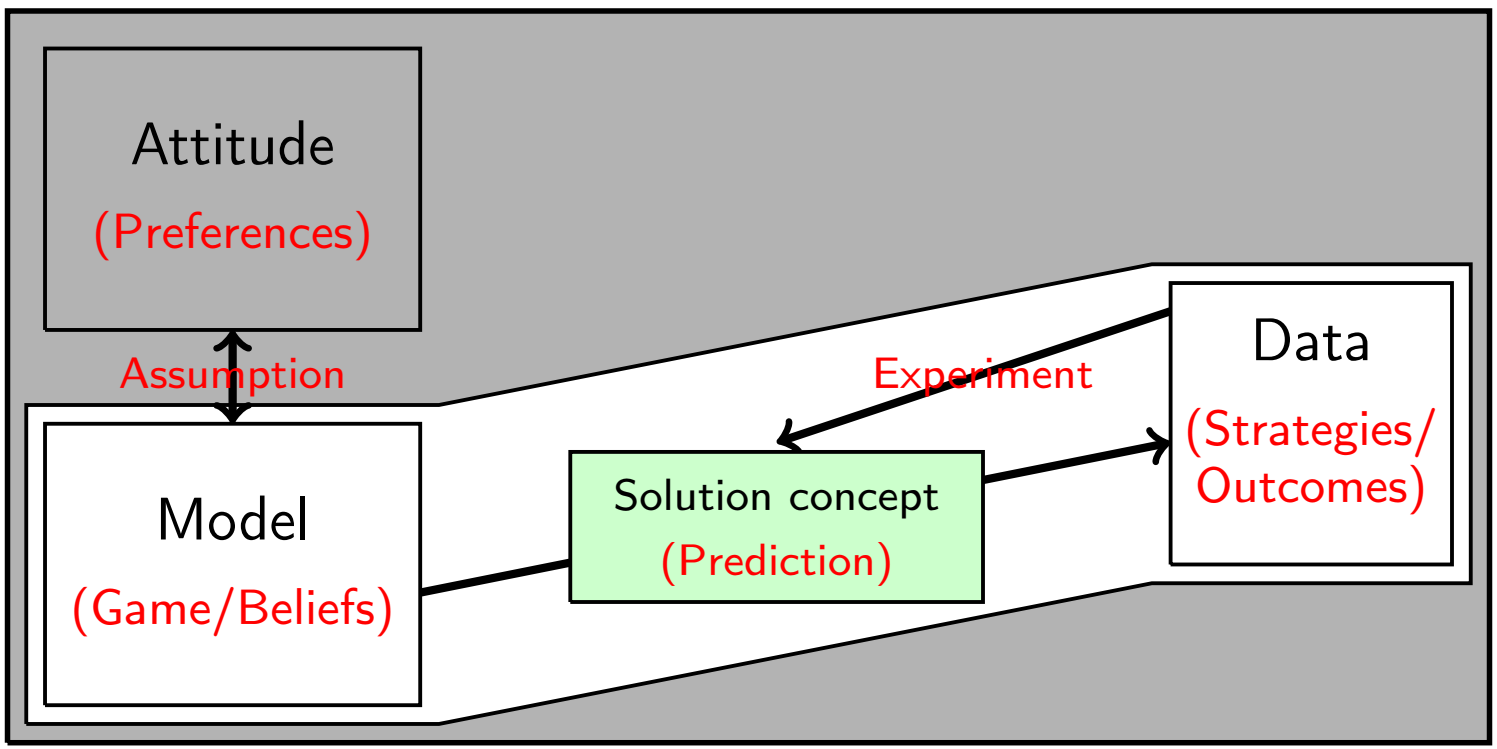
Differences to strategic games

1. Beliefs are only about future moves
2. Two types of predictions:
   - In terms of strategies (lab data)
   - In terms of outcomes (field data)



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$$u^i(s^i, s^{-i}) \geq u^i(r^i, s^{-i}), \text{ for all } r^i$$

- The definition is almost the same as in static games:
  1. Every player is rational
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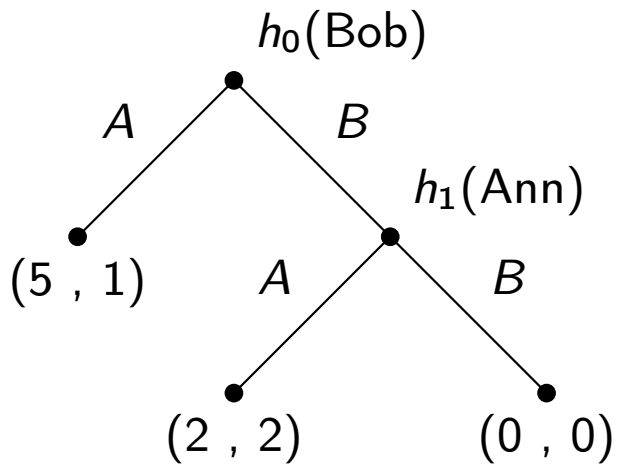
Extensive game



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	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

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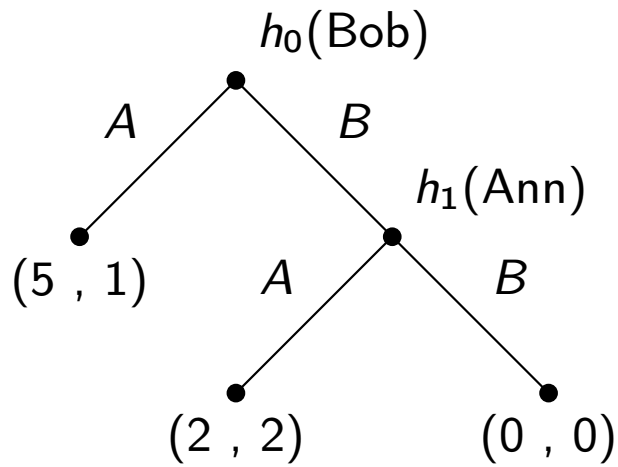

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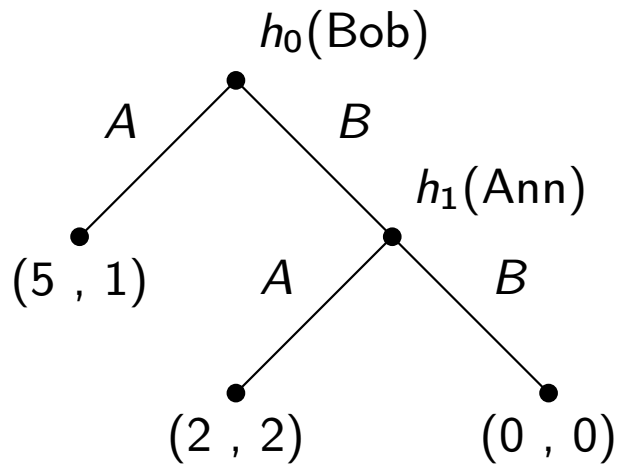

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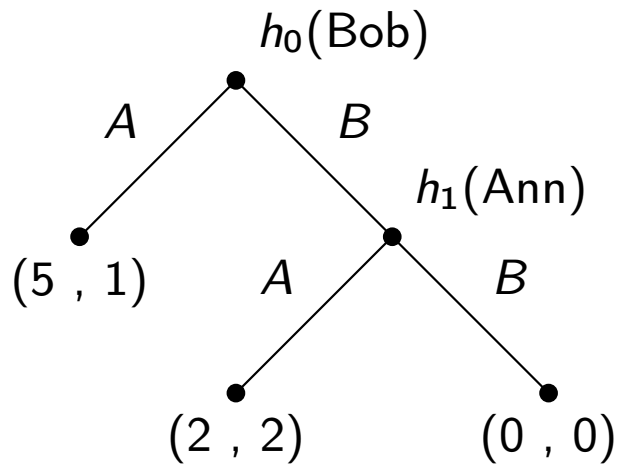

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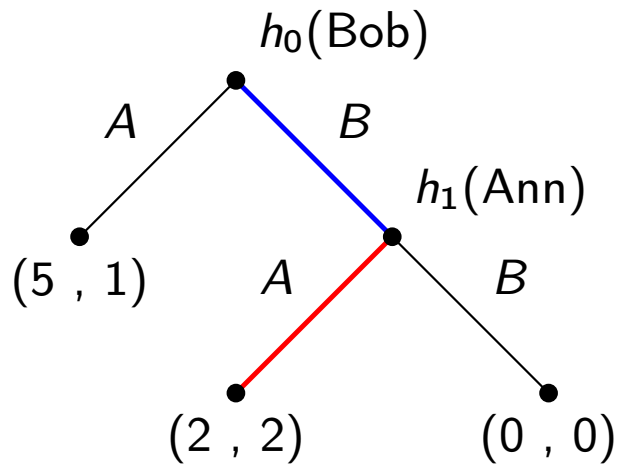

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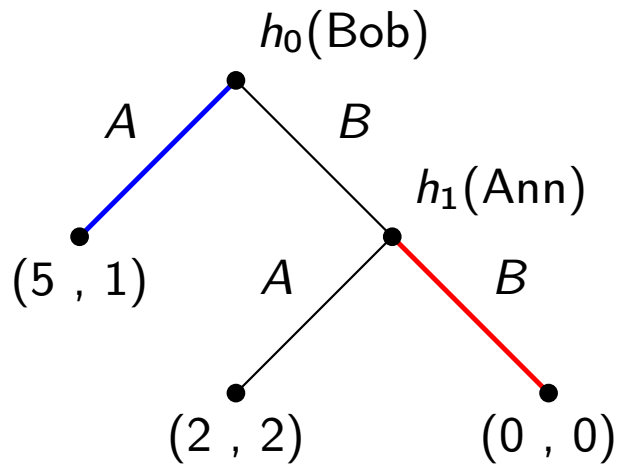

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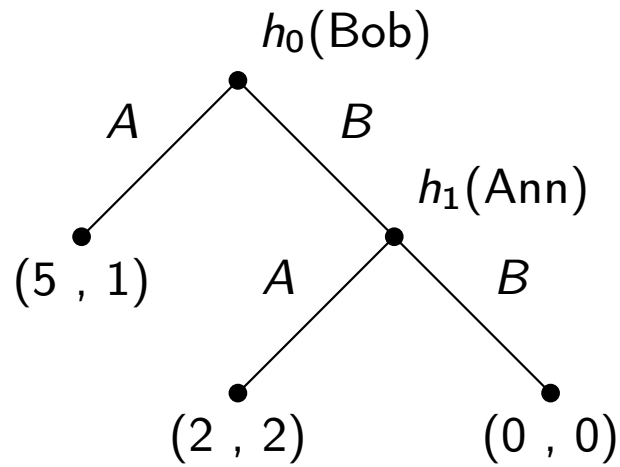

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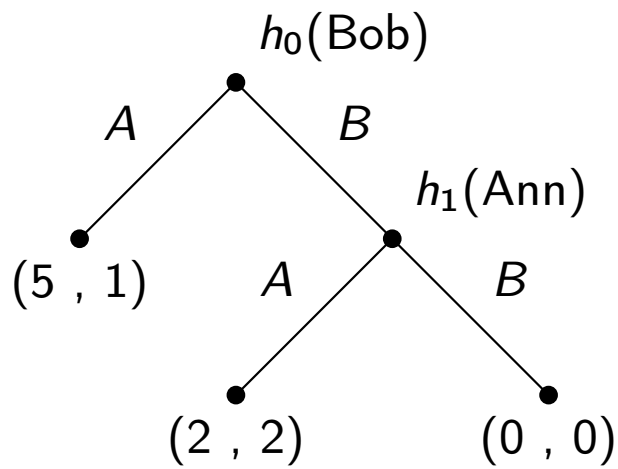
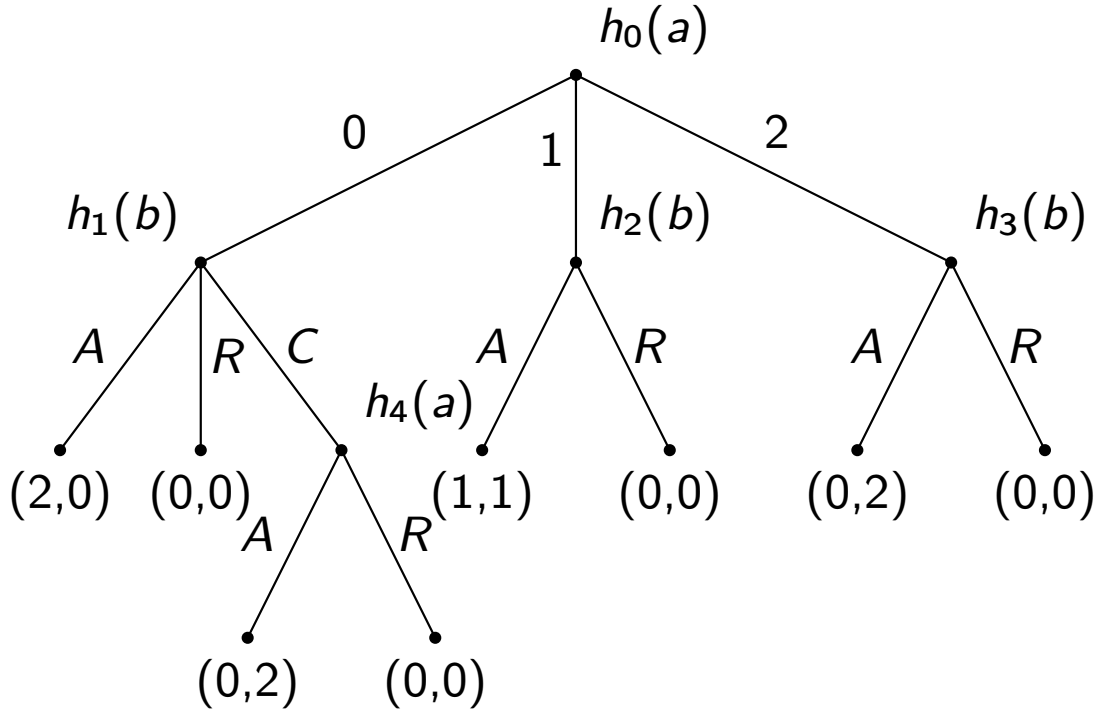
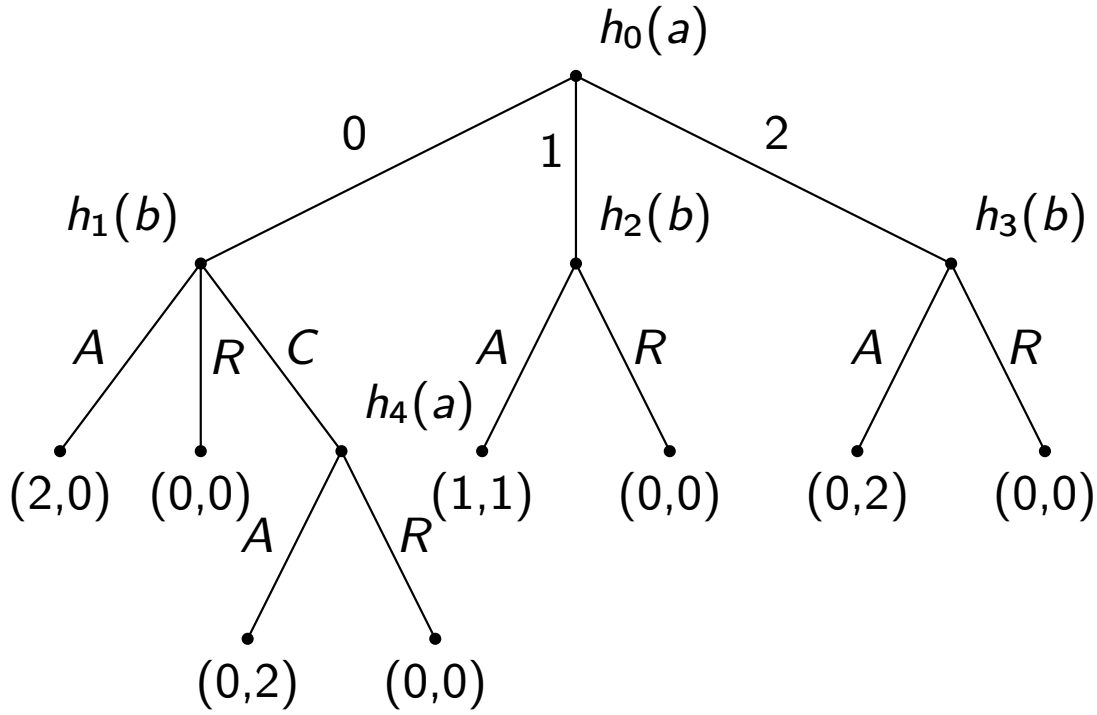
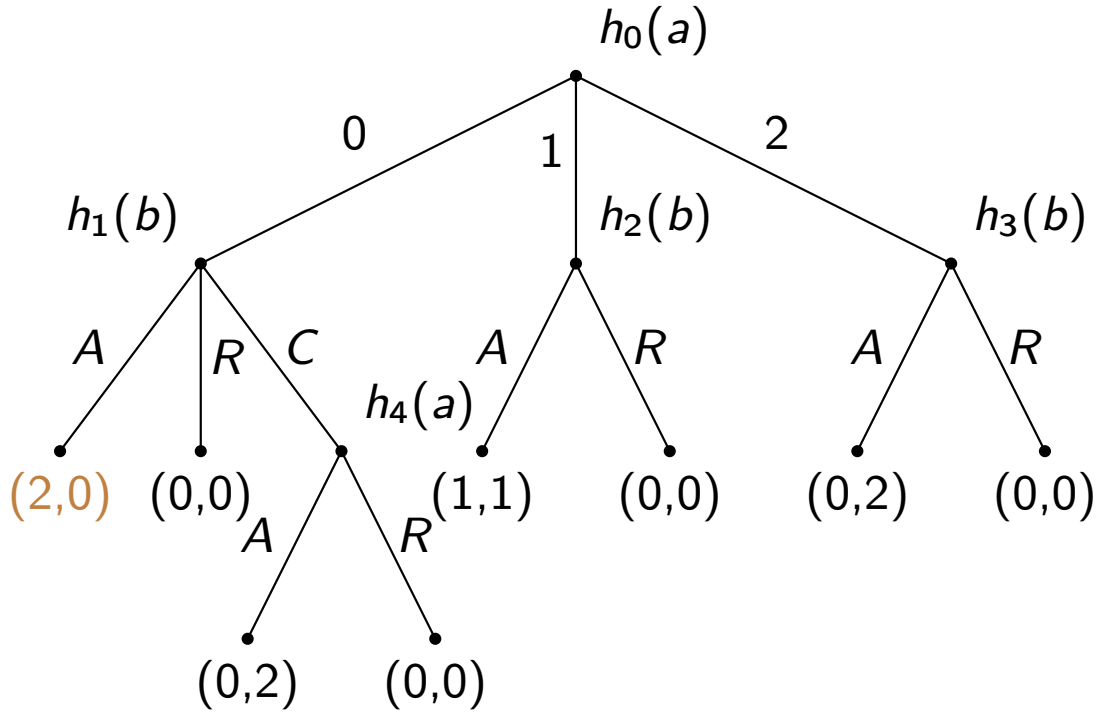




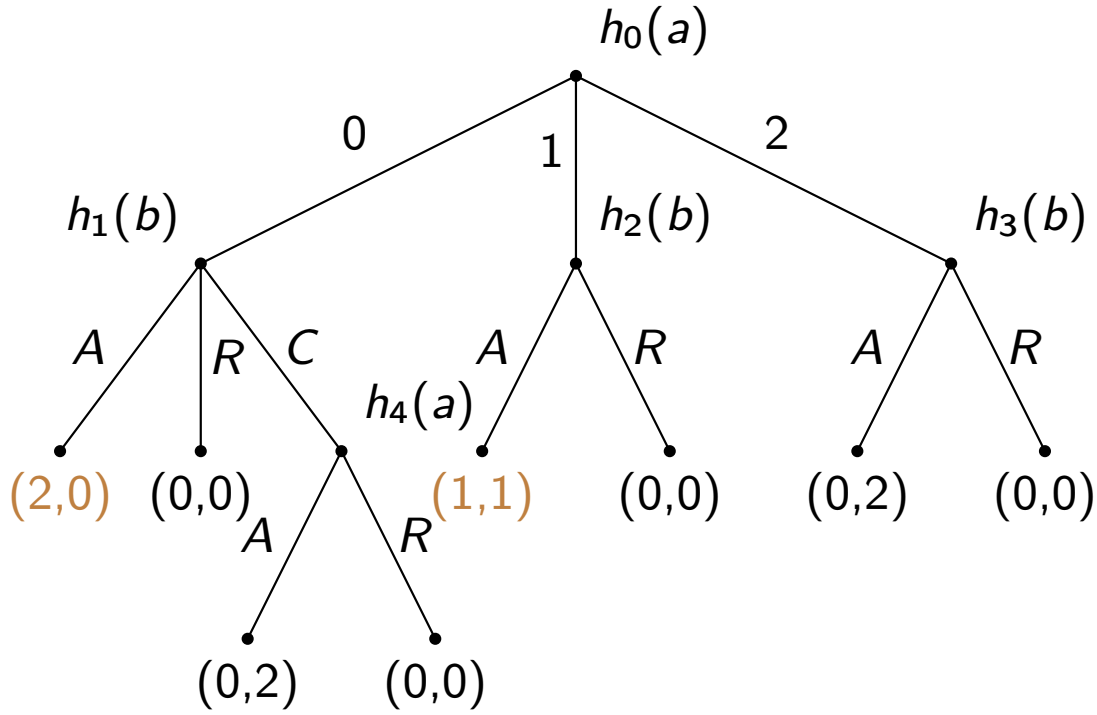
	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
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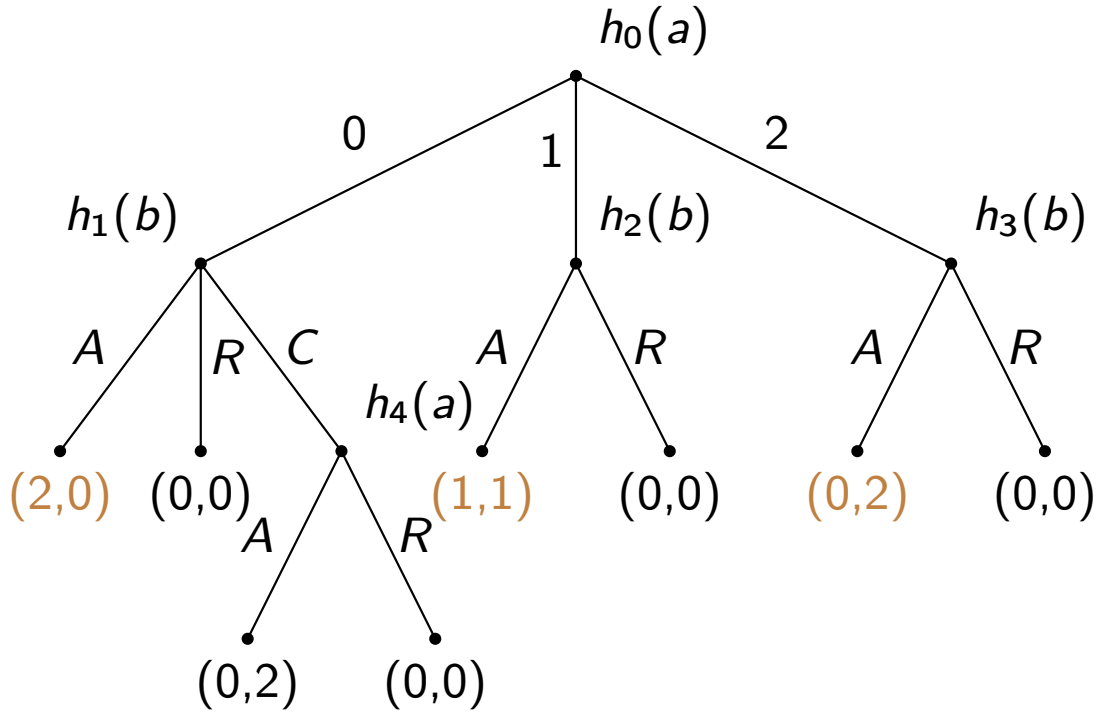
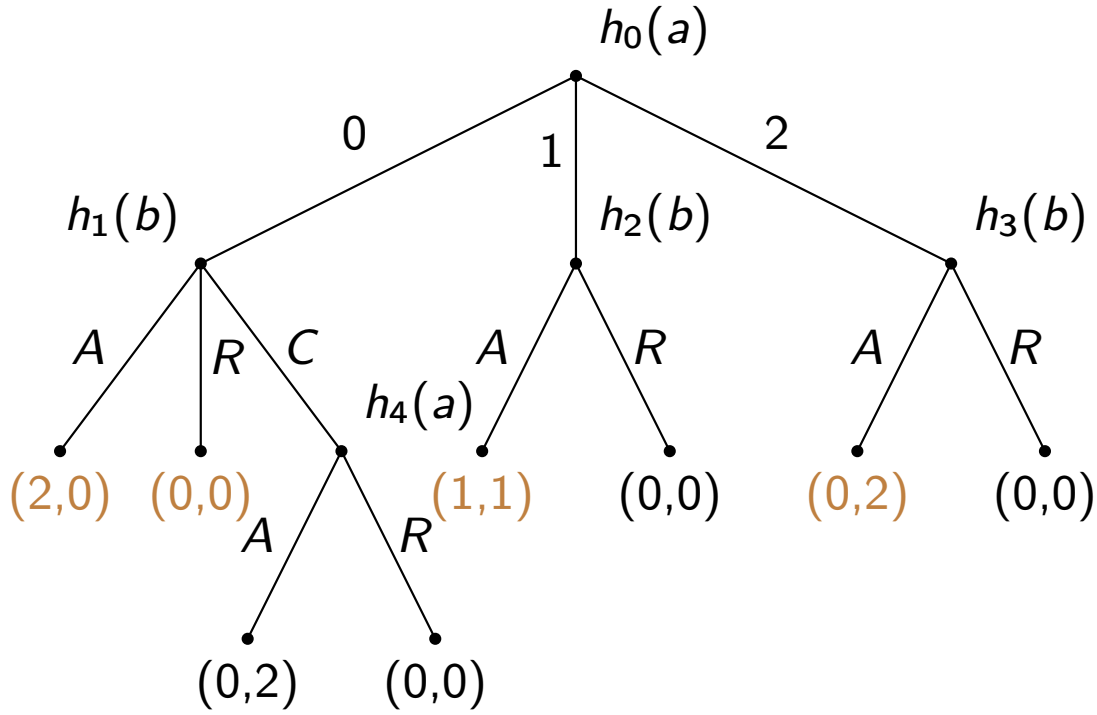
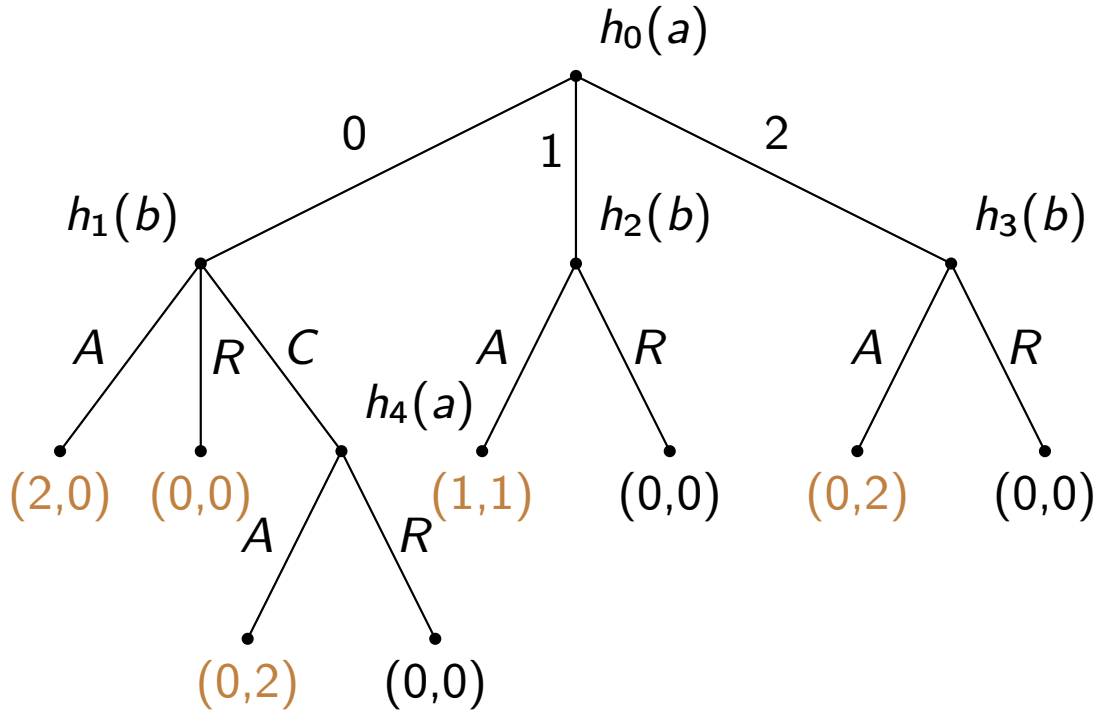


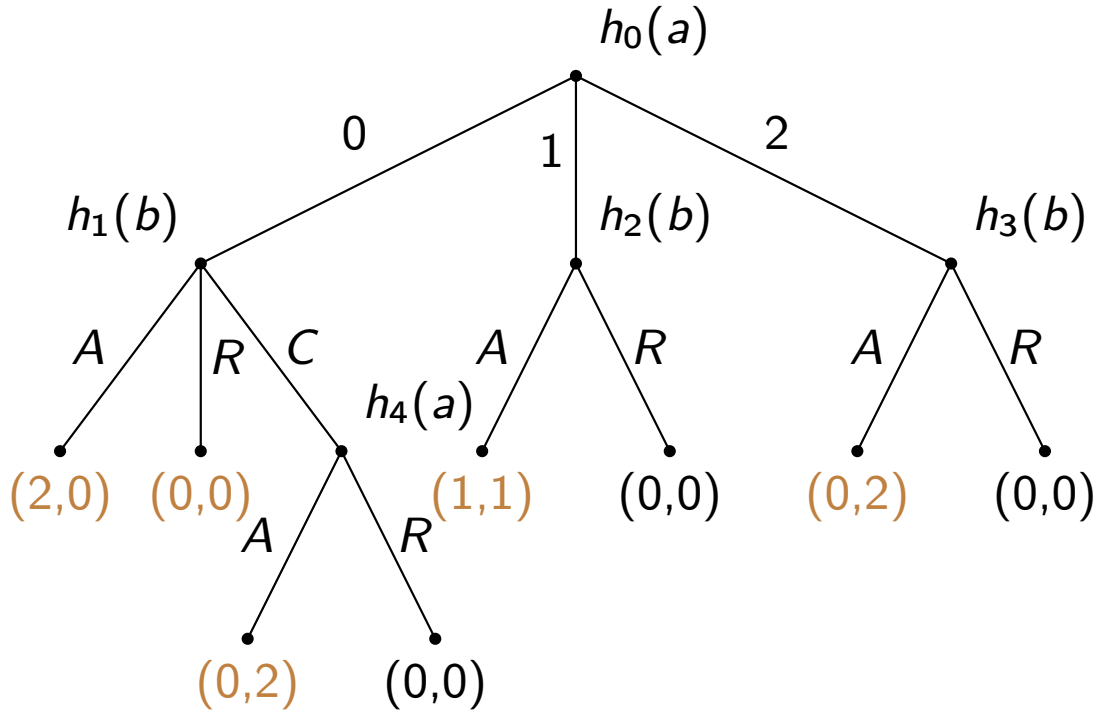
	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0

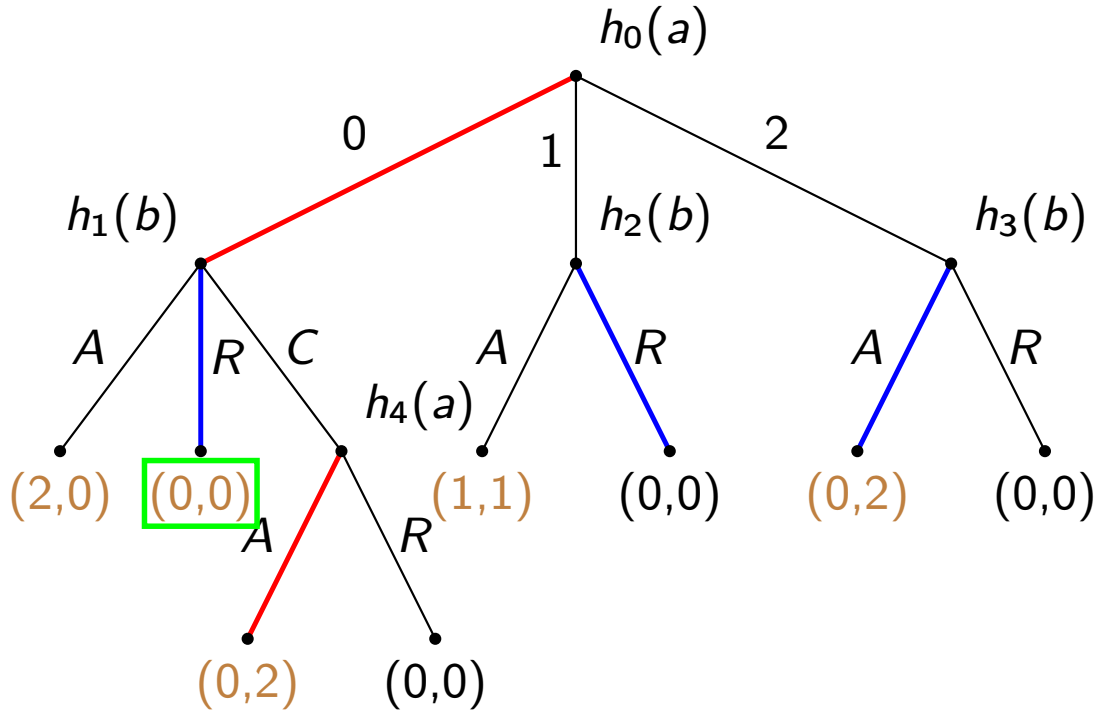


	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,2	0,2	0,2	0,0	0,0	0,2	0,2	0,2	0,2	0,2	0,0
2*R*	0,2	0,2	0,2	0,2	0,0	0,0	0,2	0,2	0,2	0,2	0,2	0,0

Nash Equilibrium outcome
does not necessarily imply
Nash Equilibrium strategy profile



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,0
2*R*	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,0

Nash Equilibrium outcome
does not necessarily imply
Nash Equilibrium strategy profile

- Nash Equilibrium assumes that

  players check rationality only at the beginning of the game

- At later instances, they follow their plan even if this is stupid

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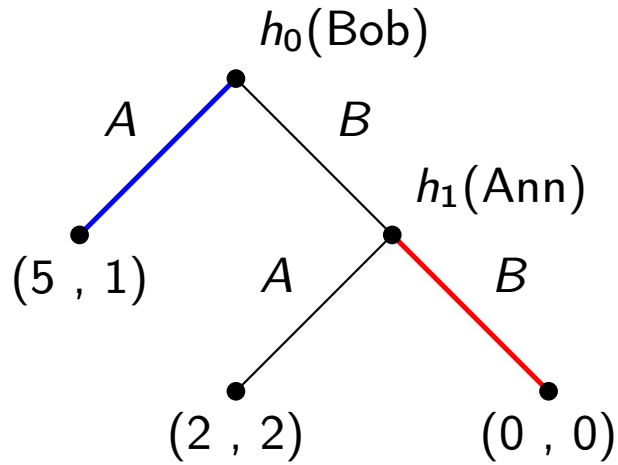
  1. Every player is rational at the beginning of the game
  2. Every player has correct beliefs at the beginning of the game

Strategic game

	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

Extensive game



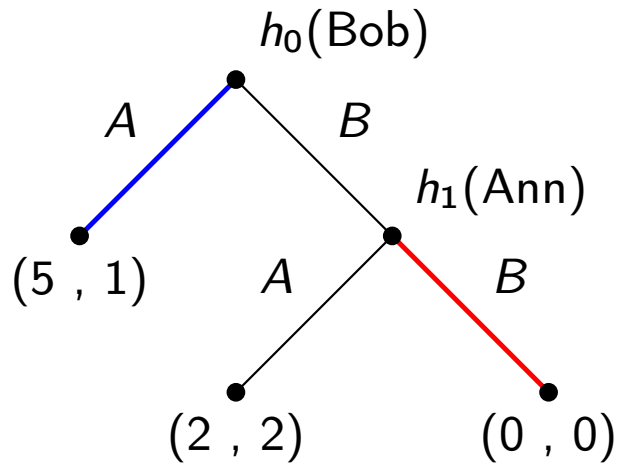
- Nash Equilibrium assumptions:
  1. Every player is rational at the beginning of the game
  2. Every player has correct beliefs at the beginning of the game
- At h_1 Ann is choosing irrationally!!!

Is this NE a reasonable prediction?

Strategic game

	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

Extensive game



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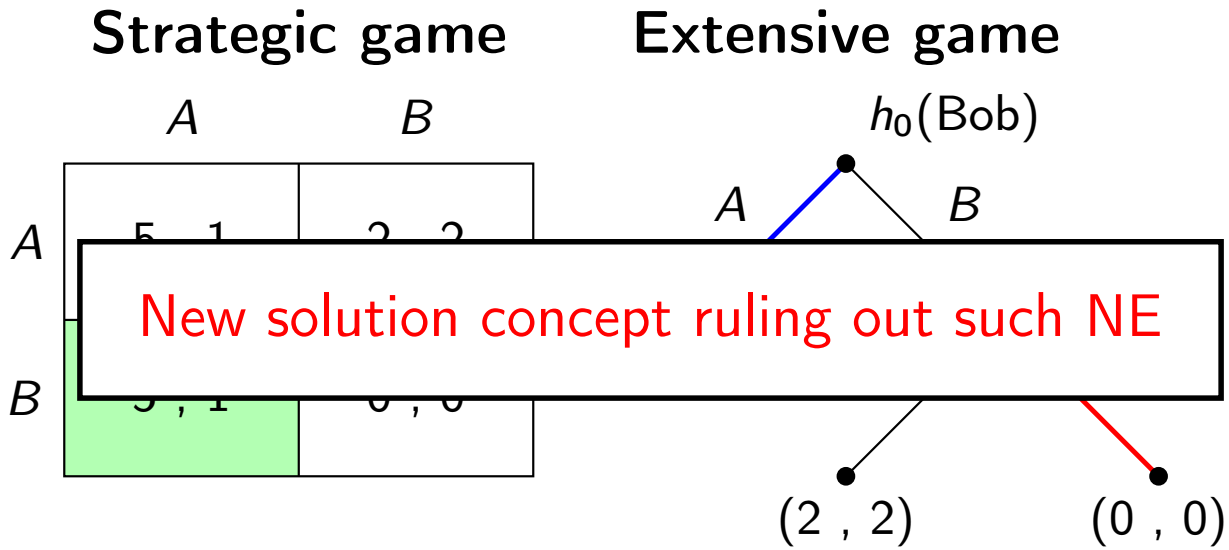
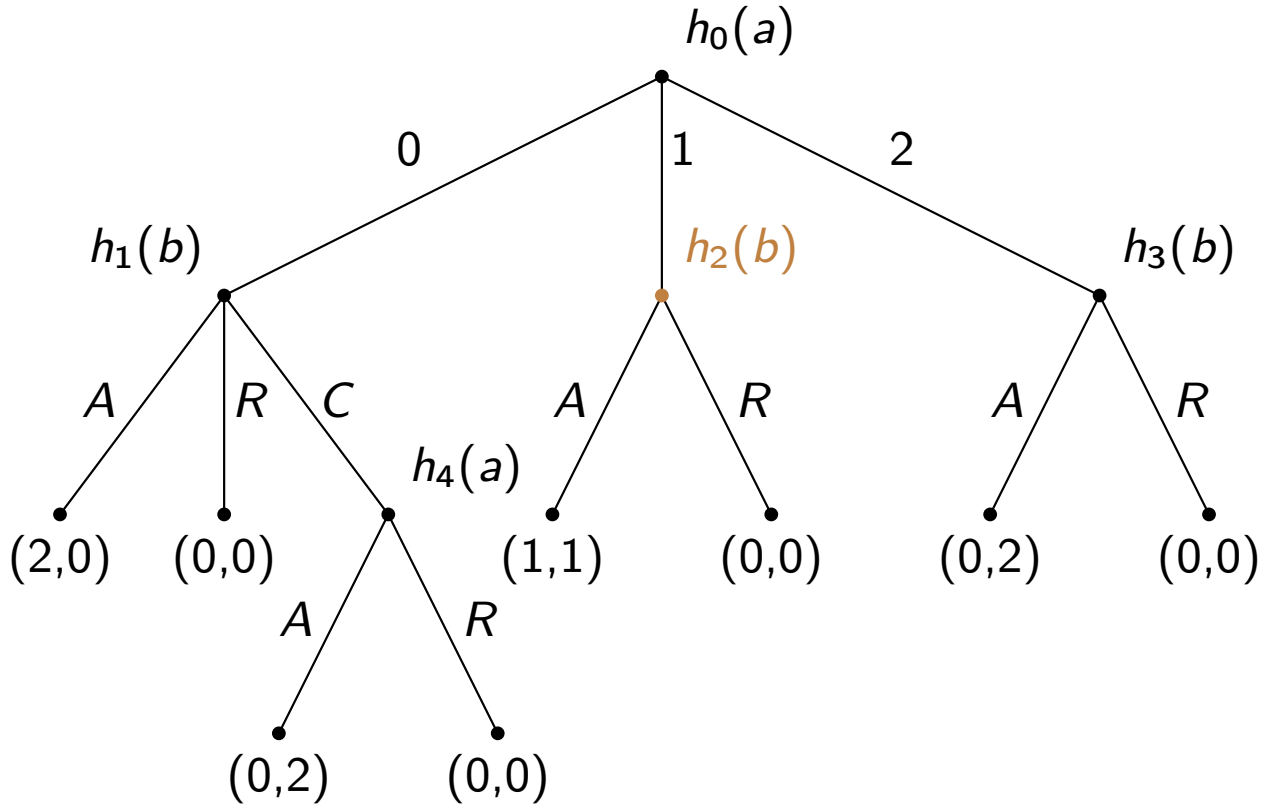


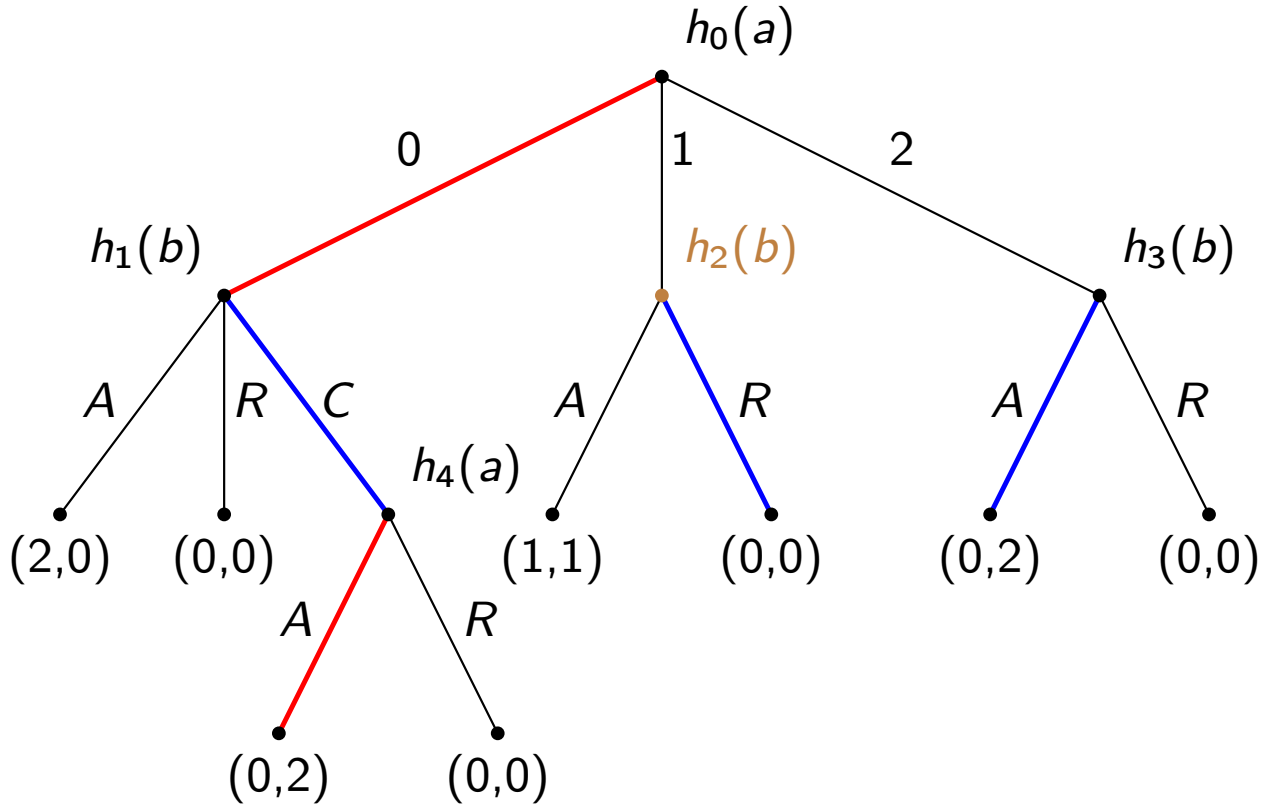
A strategy profile leads to a unique **terminal node** $z(s, h)$ and thus induces utility $u_i(s, h)$ if followed from h onwards



$$S^a = \{0A, 0R, 1A, 1R, 2A, 2R\}$$

$$S^b = \{AAA, AAR, ARA, ARR, RAA, RAR, RRA, RRR, CAA, CAR, CRA, CRR\}$$

A strategy profile leads to a unique **terminal node** $z(s, h)$ and thus induces utility $u_i(s, h)$ if followed from h onwards



$$S^a = \{0A, 0R, 1A, 1R, 2A, 2R\}$$

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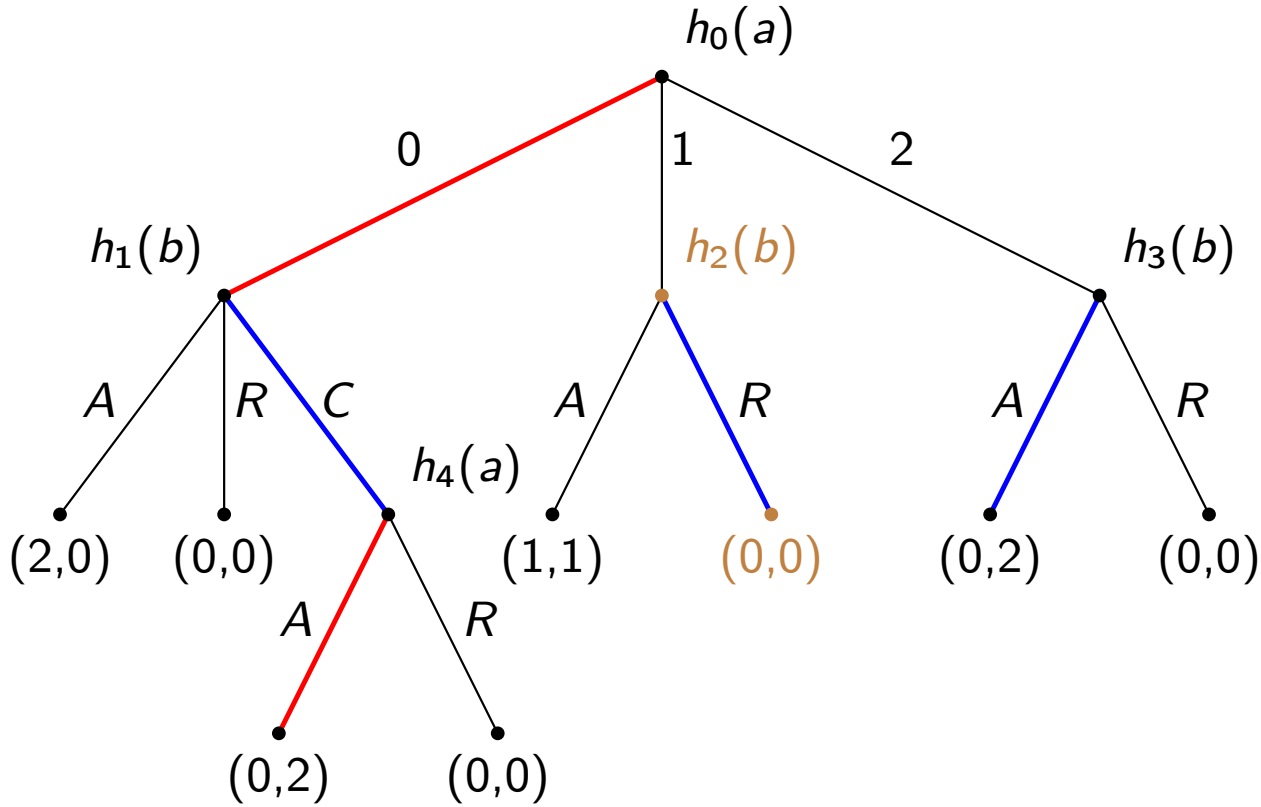


$$S^a = \{0A, 0R, 1A, 1R, 2A, 2R\}$$

$$S^b = \{AAA, AAR, ARA, ARR, RAA, RAR, RRA, RRR, CAA, CAR, CRA, CRR\}$$

Definition

A strategy profile $s = (s^1, \ldots, s^n)$ is a **Subgame Perfect Equilibrium (SPE)** if for each player i and each h with $P(h) = i$:

$$u^i(s^i, s^{-i}, h) \geq u^i(r^i, s^{-i}, h), \text{ for all } r^i$$

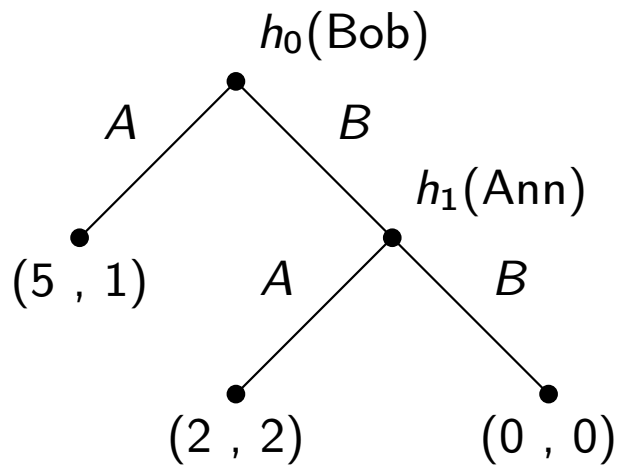
- The definition requires:
  1. Every player is rational throughout the game
  2. Every player has correct beliefs throughout the game
- Every SPE is a NE (equilibrium refinement):

  "throughout the game" $\Rightarrow$ "beginning of the game"

Strategic game

	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

Extensive game



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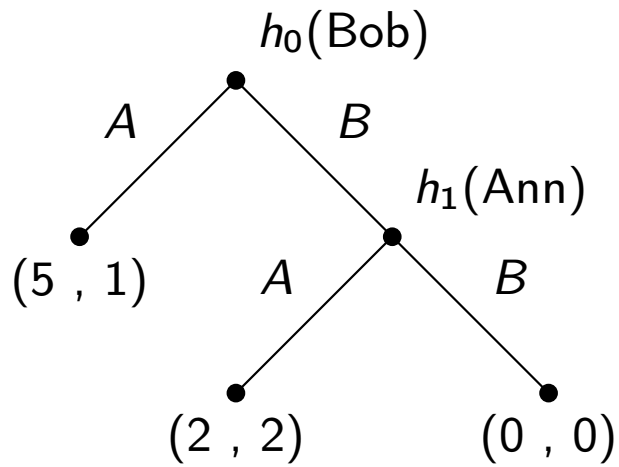
Strategic game

	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

Extensive game



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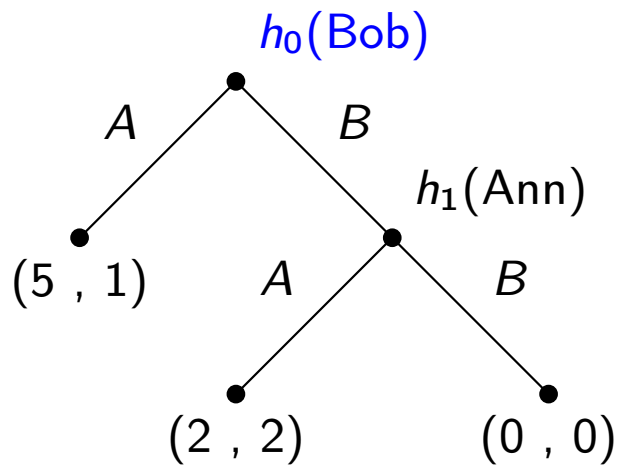
Strategic game

	A	B
A	5 , 1	2 , 2
B	5 , 1	0 , 0

Extensive game



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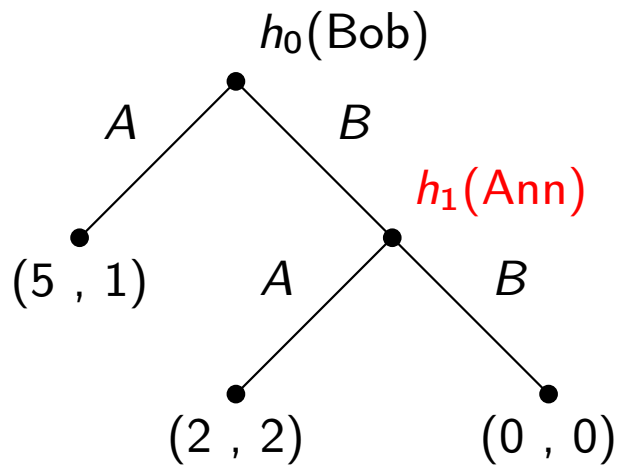
Strategic game

	A	B
A	2 , 2	2 , 2
B	0 , 0	0 , 0

Extensive game



Definition

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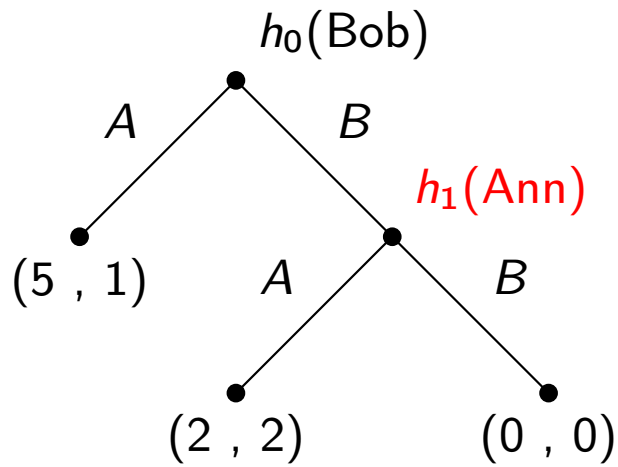
Strategic game

	A	B
A	2 , 2	2 , 2
B	0 , 0	0 , 0

Extensive game



Definition

A strategy profile $s = (s^1, \ldots, s^n)$ is a **Subgame Perfect Equilibrium (SPE)** if for each player i and each h with $P(h) = i$:

$$u^i(s^i, s^{-i}, h) \geq u^i(r^i, s^{-i}, h), \text{ for all } r^i$$

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  2. Every player has correct beliefs throughout the game
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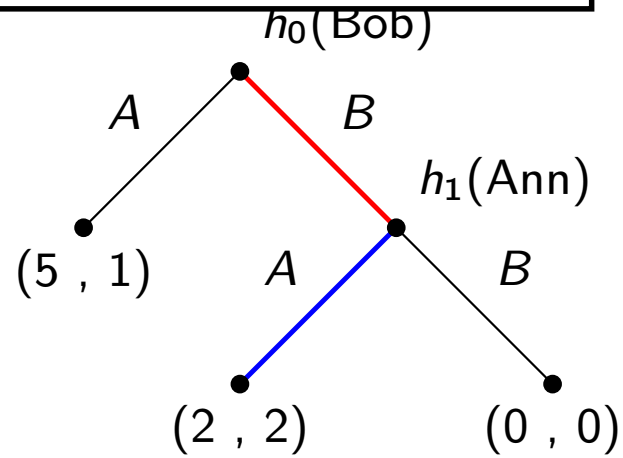
  "throughout the game" $\Rightarrow$ "beginning of the game"

The only SPE is A for Ann and B for Bob

	A	B
A	2 , 2	2 , 2
B	0 , 0	0 , 0



Definition

We split the nodes in different groups:

- **Group 0:** terminal nodes
- **Group 1:** only followed by nodes in Group 0
- **Group 2:** only followed by nodes in Group 0, Group 1

  ⋮ ⋮ ⋮

- **Group n:** only followed by nodes in Group 0, ..., Group $n-1$

Definition

A strategy profile survives **Backward Induction (BI)** if it prescribes actions that satisfy the following steps:

- **Step 1:** rational actions are picked in all nodes in Group 1
- **Step 2:** rational actions are picked in all nodes in Group 2, given the actions that have been picked in Step 1

  ⋮ ⋮ ⋮

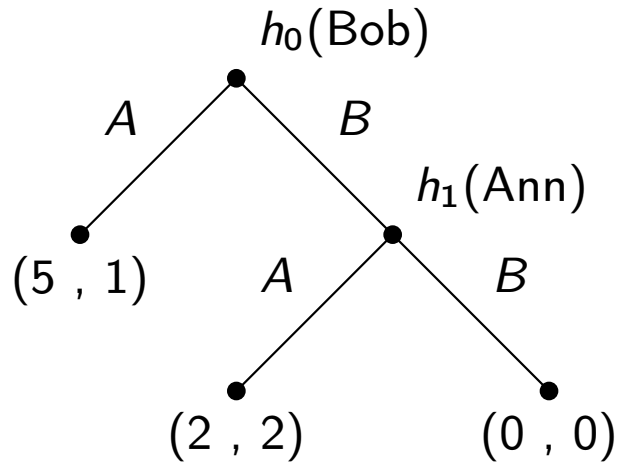
- **Step n:** rational actions are picked in all nodes in Group n, given the actions that have been picked in Steps $1, \ldots, n-1$

Theorem

A strategy profile in an extensive game is a SPE

if and only if

it survives Backward Induction



Group 1 nodes =

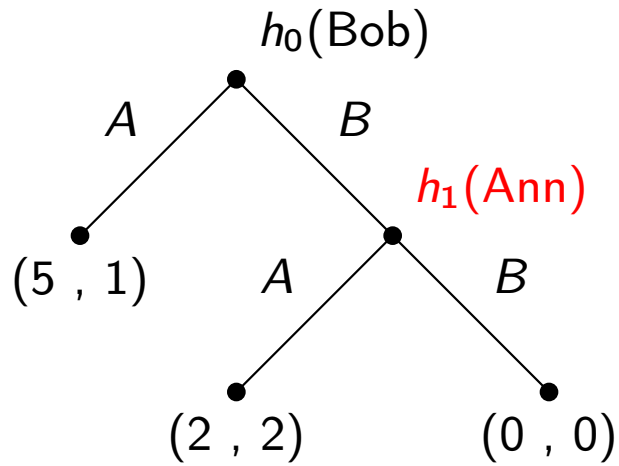
Group 2 nodes =

Theorem

A strategy profile in an extensive game is a SPE

if and only if

it survives Backward Induction



Group 1 nodes $= \{h_1\}$

Group 2 nodes $=$

Theorem

A strategy profile in an extensive game is a SPE

if and only if

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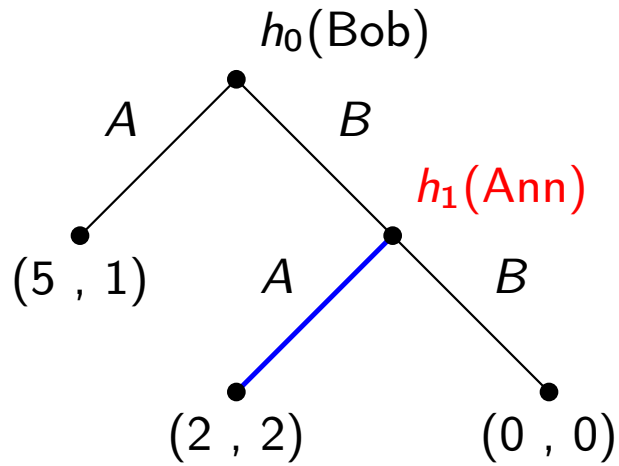

Group 1 nodes $= \{h_1\}$

Group 2 nodes $=$

Theorem

A strategy profile in an extensive game is a SPE

if and only if

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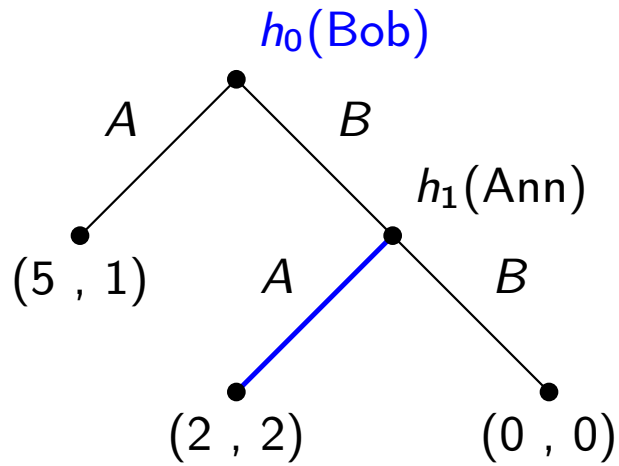


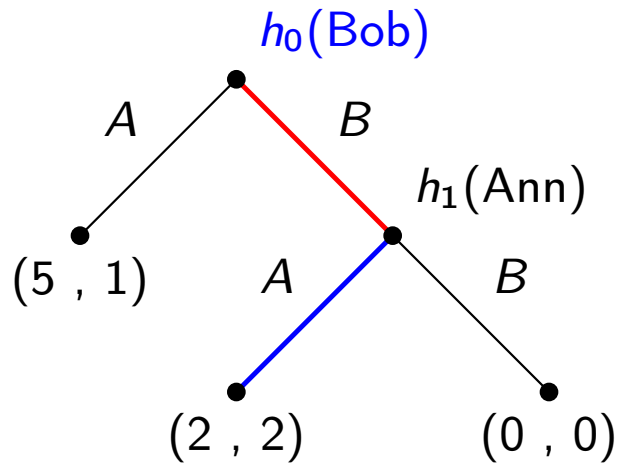
$$\text{Group 1 nodes} = \{h_1\}$$
$$\text{Group 2 nodes} = \{h_0\}$$

Theorem

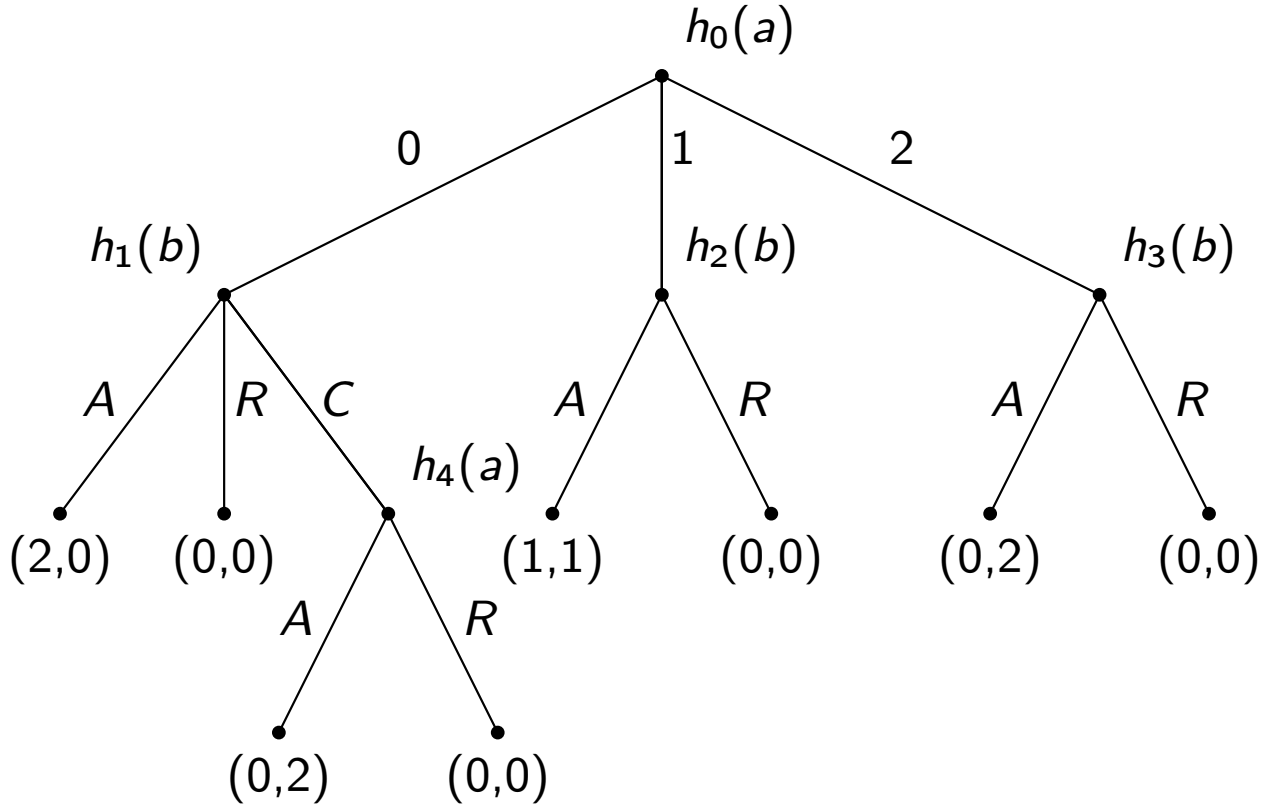
A strategy profile in an extensive game is a SPE

if and only if

it survives Backward Induction



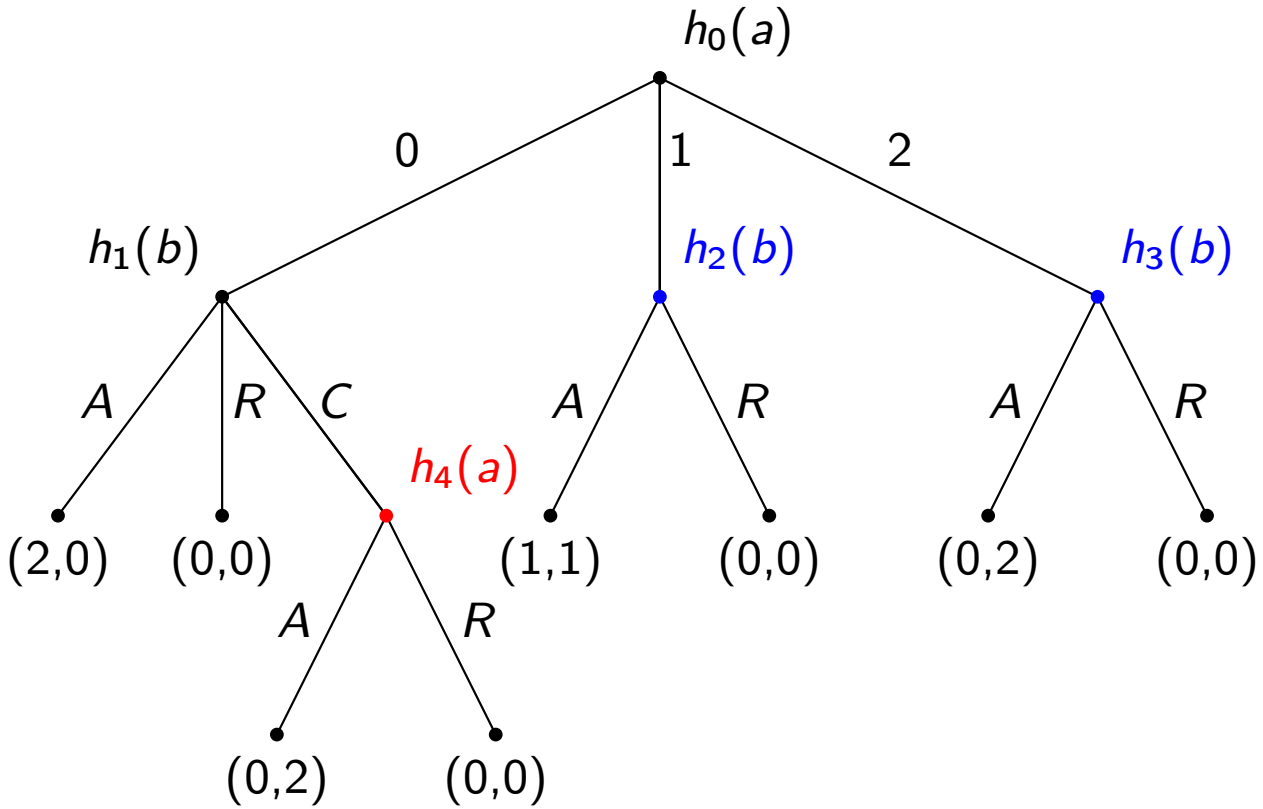
$$\text{Group 1 nodes} = \{h_1\}$$
$$\text{Group 2 nodes} = \{h_0\}$$



Group 1 nodes =

Group 2 nodes =

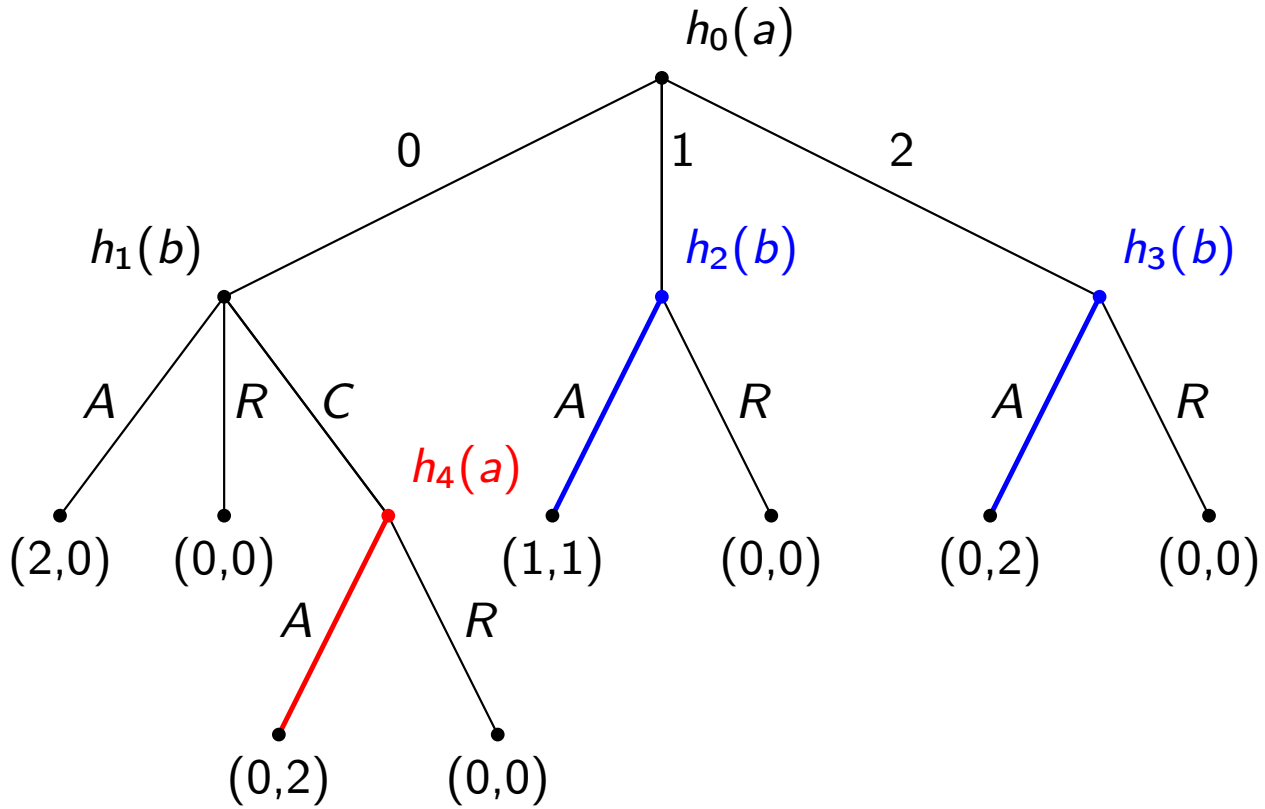
Group 3 nodes =



Group 1 nodes $=$ $\{h_2, h_3, h_4\}$

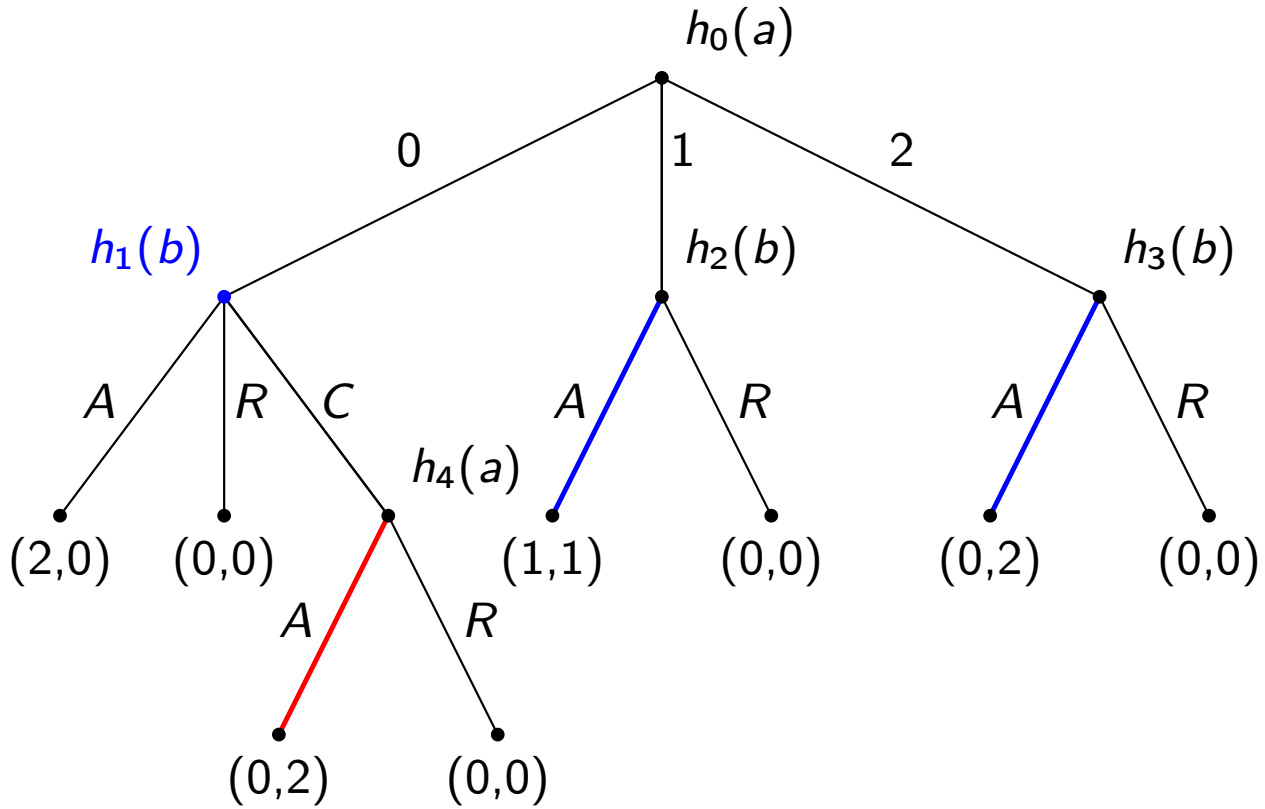
Group 2 nodes $=$

Group 3 nodes $=$



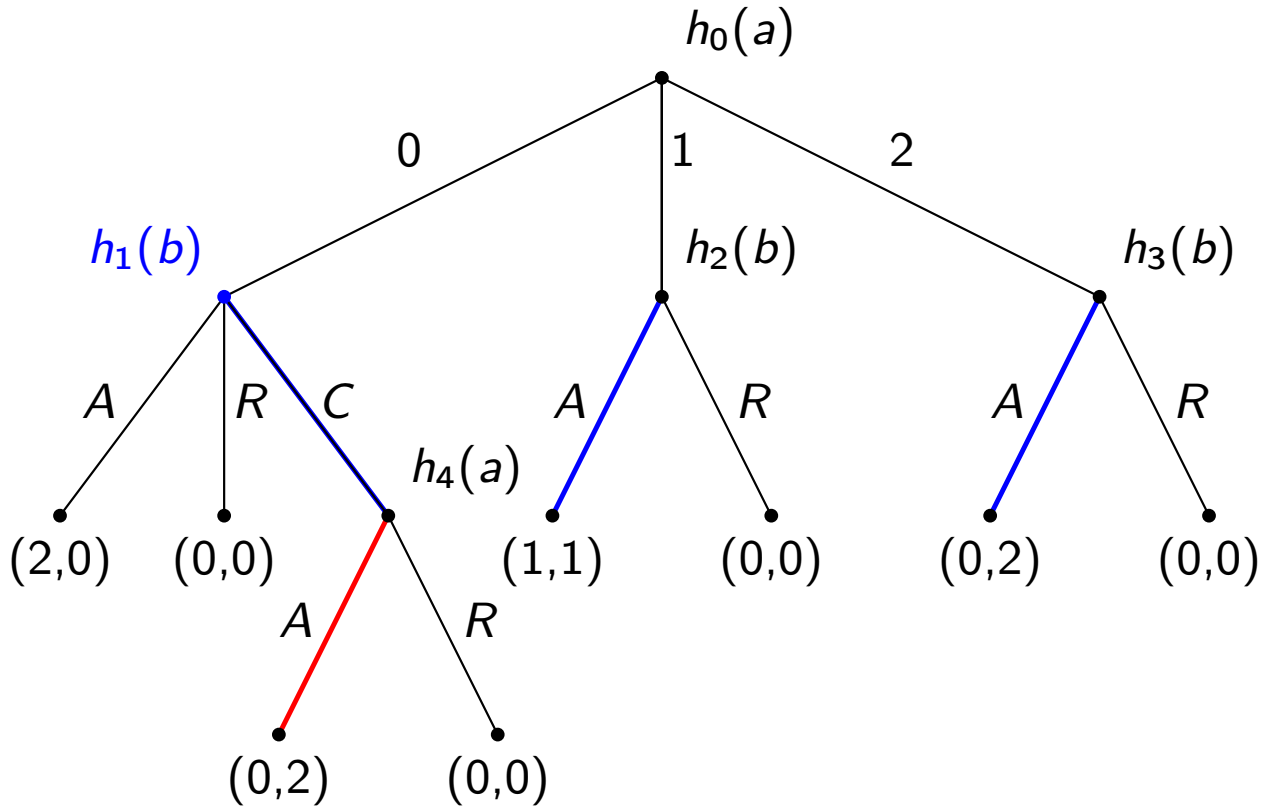
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \\ \text{Group 3 nodes} &= \end{aligned}$$

1. Ann chooses A at h_4:



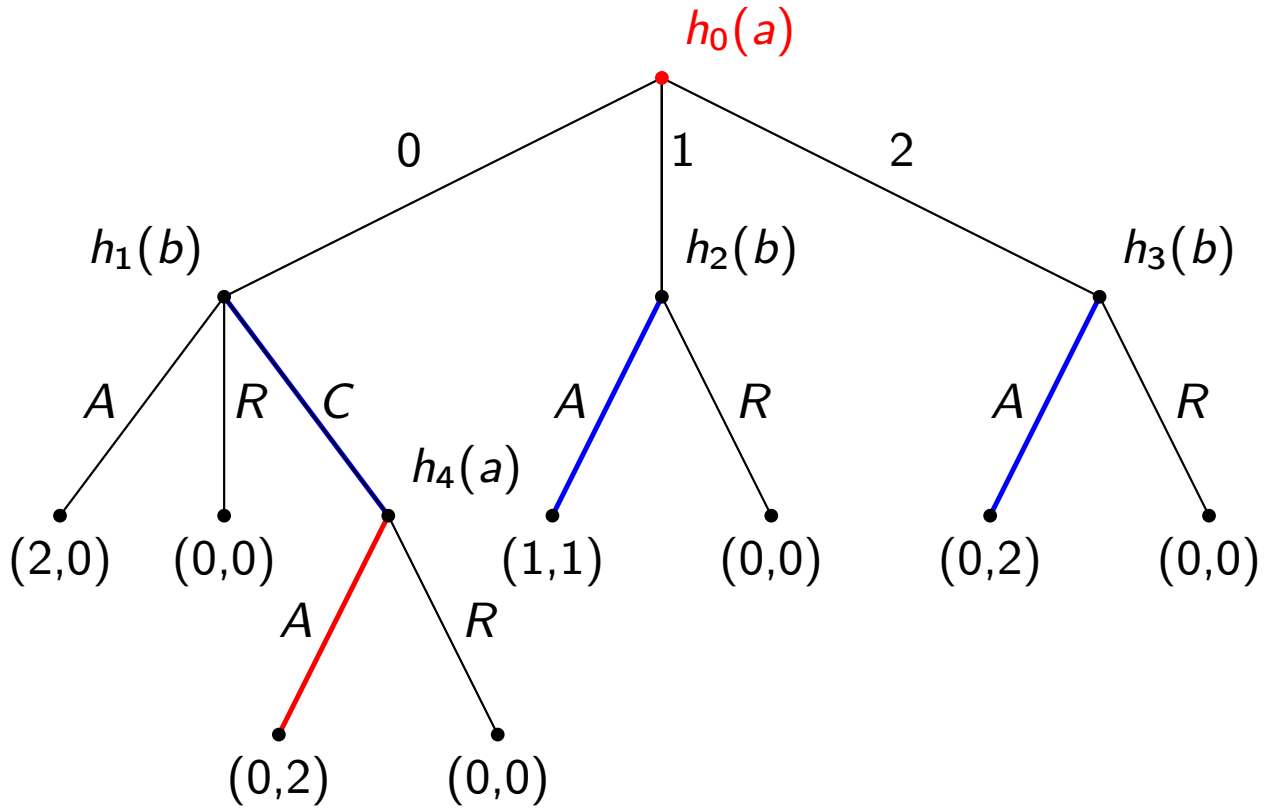
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \end{aligned}$$

1. Ann chooses A at h_4:



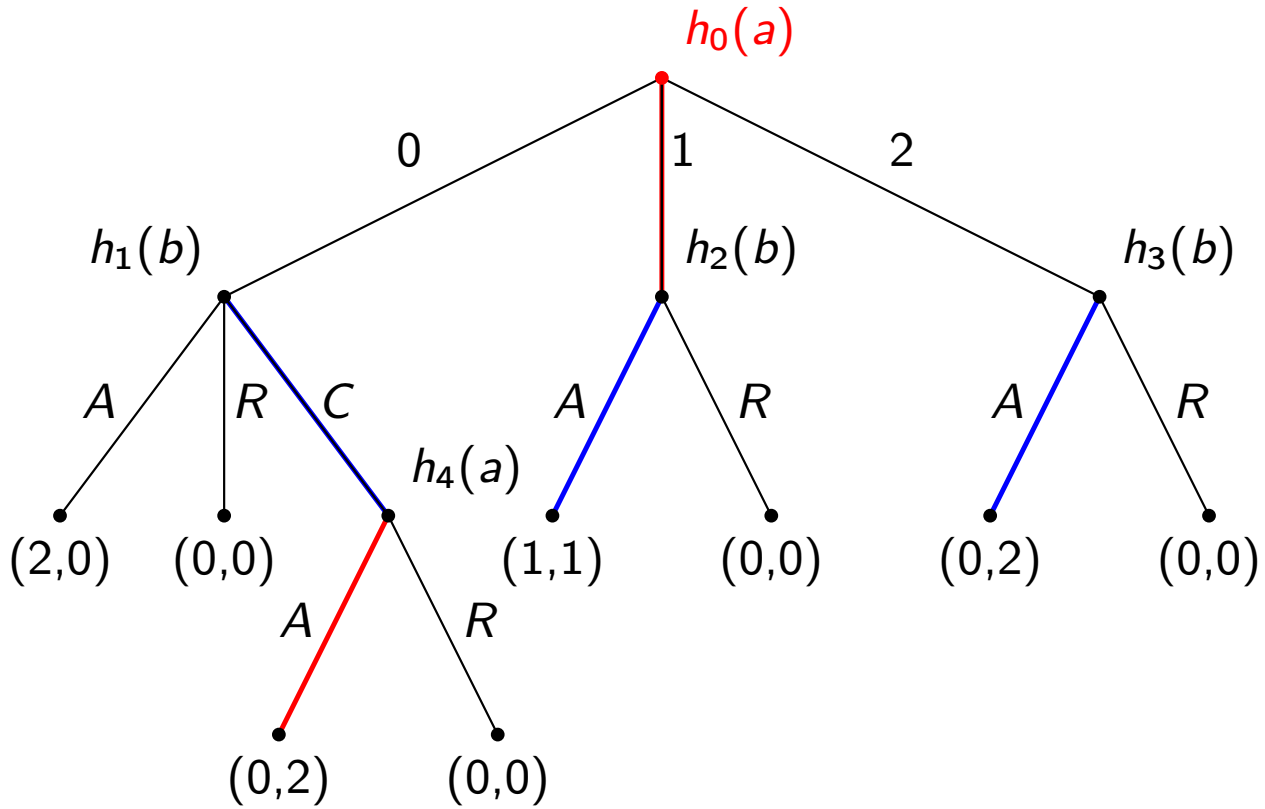
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \end{aligned}$$

1. Ann chooses A at h_4:



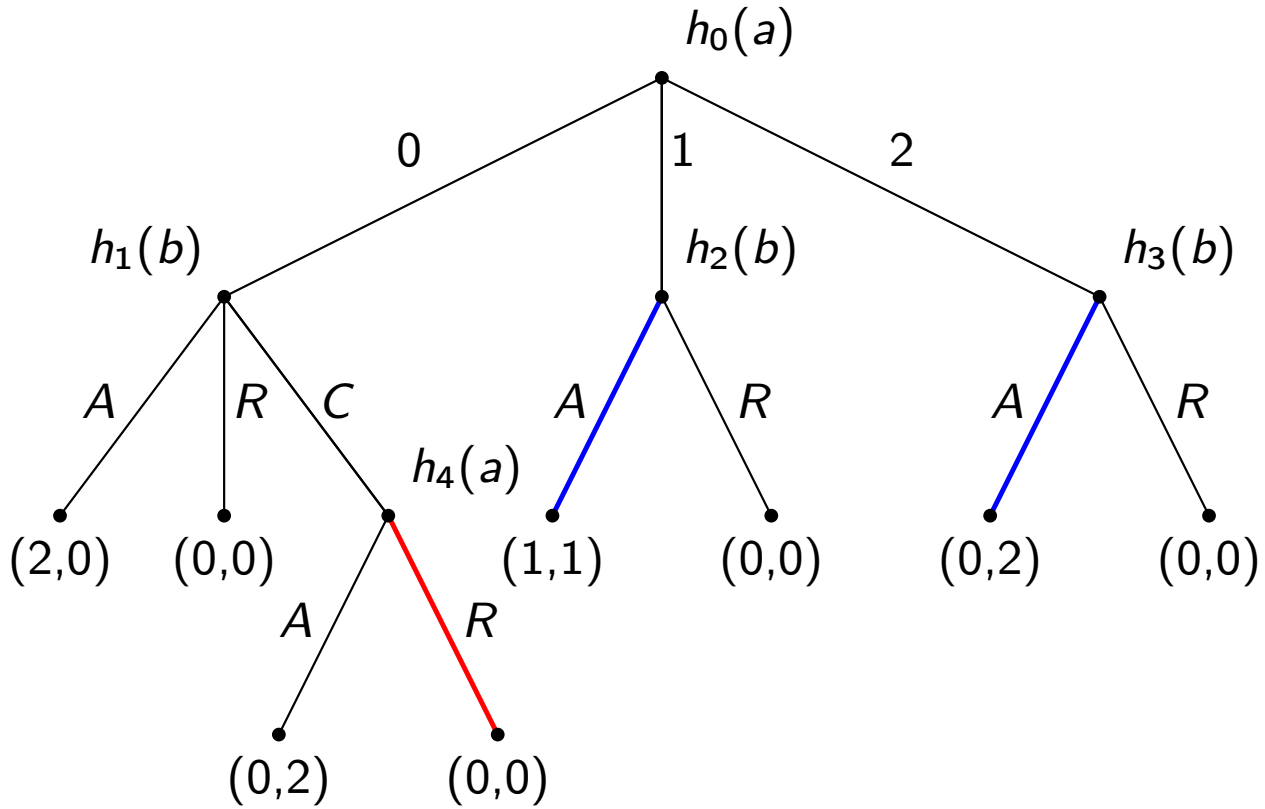
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4:



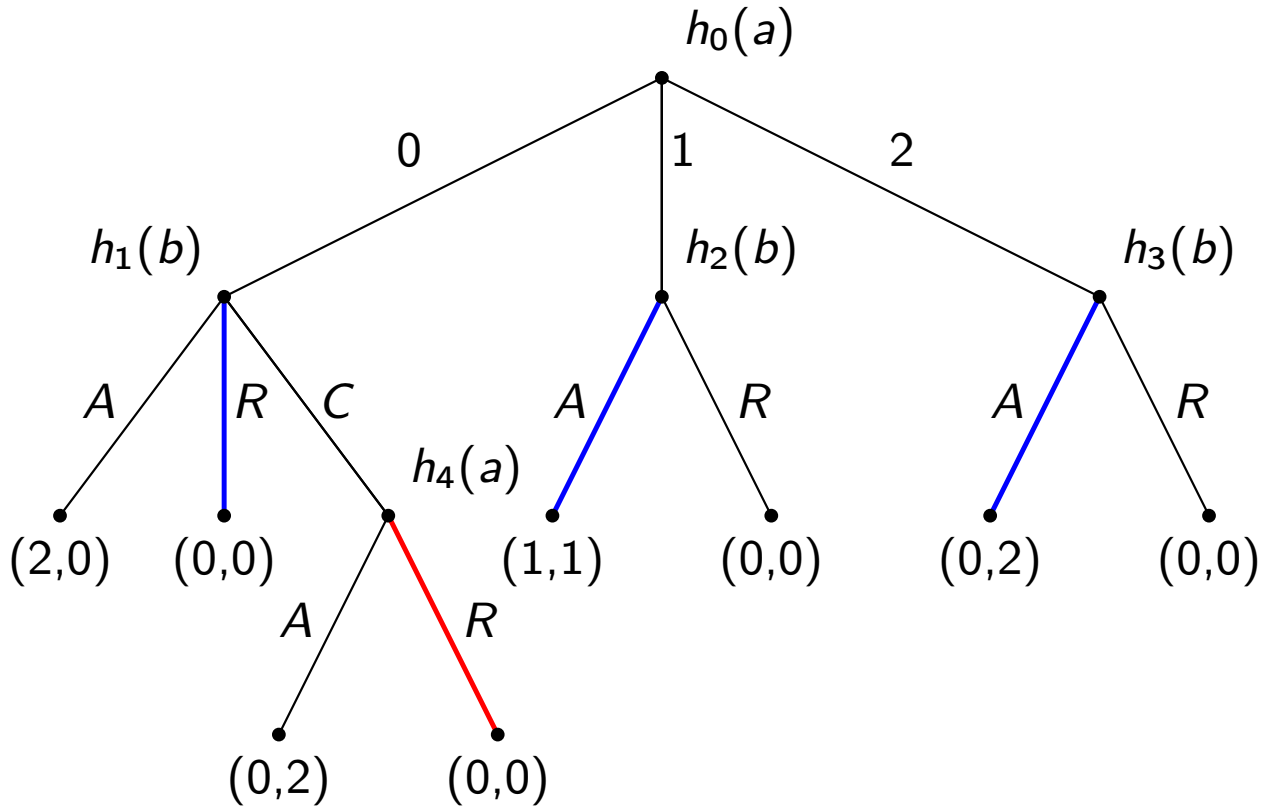
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$



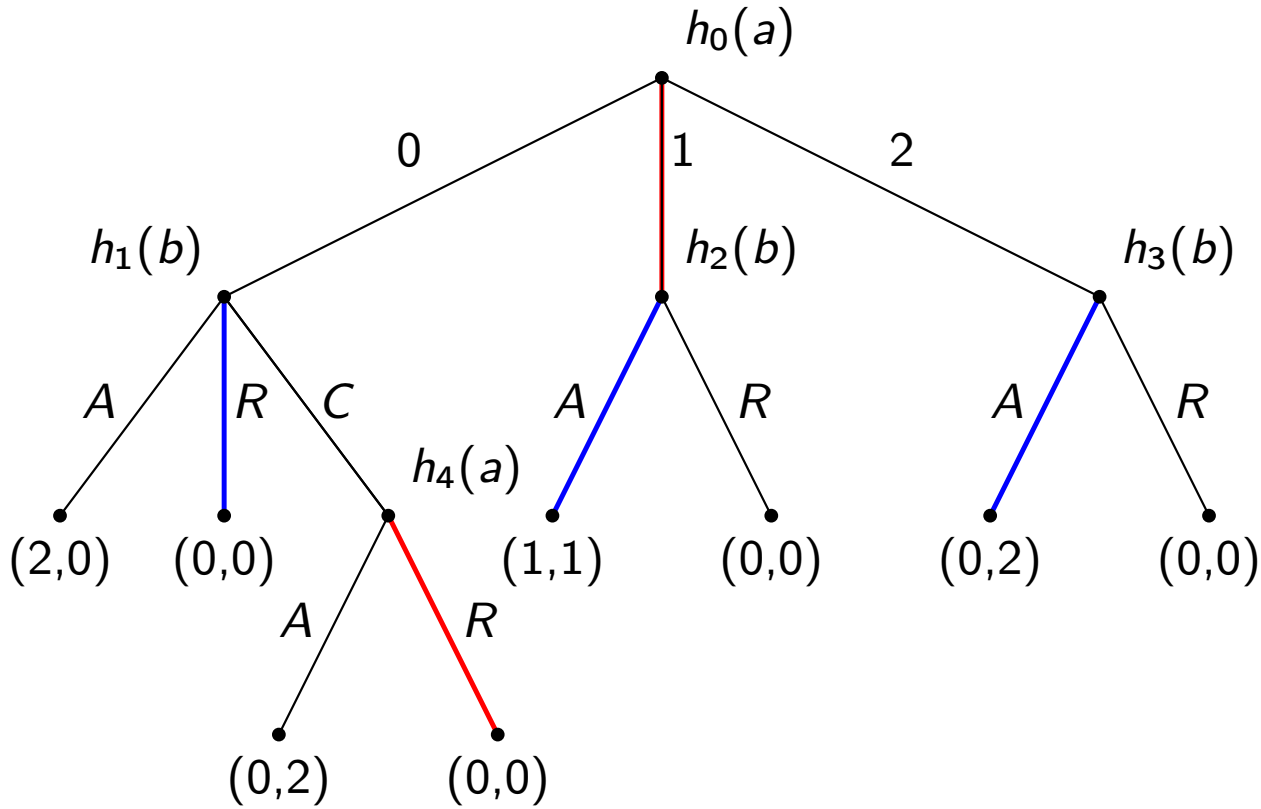
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:



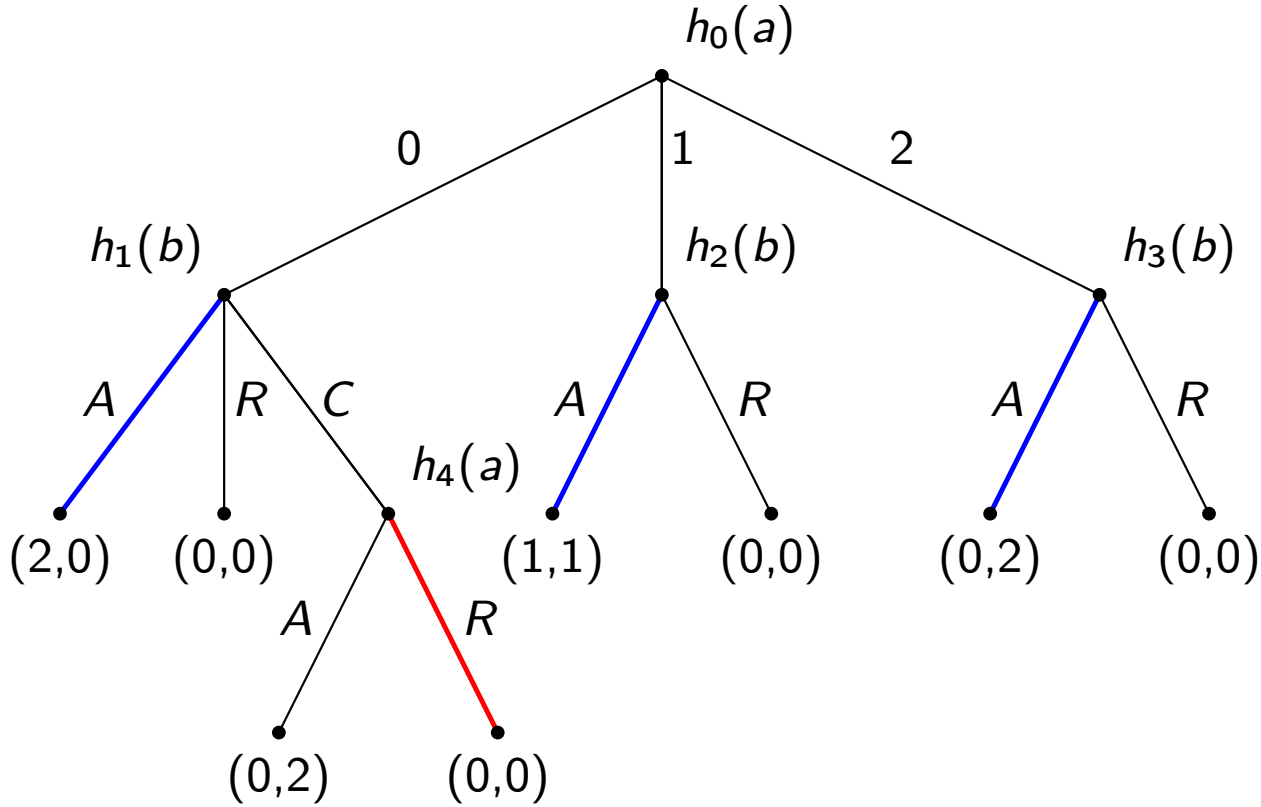
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
    1. Bob chooses R at h_1:



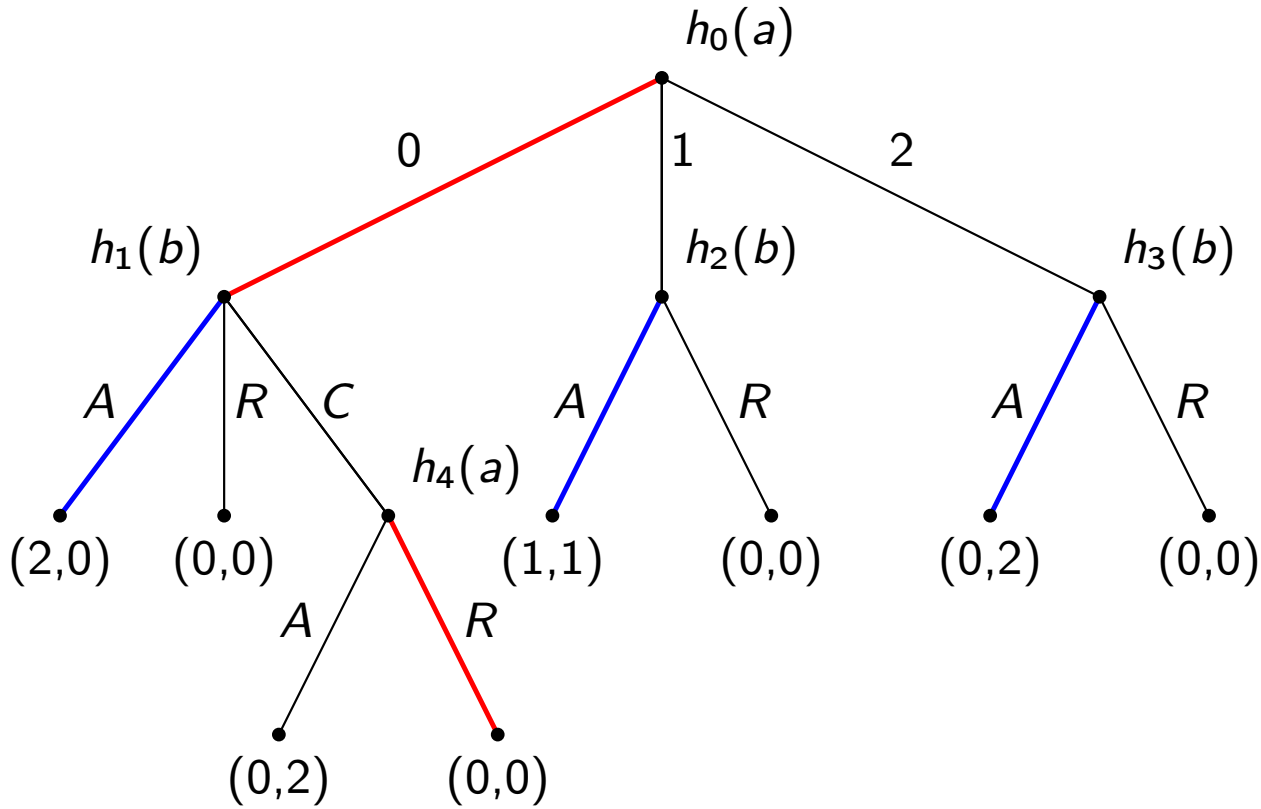
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
    1. Bob chooses R at h_1: The SPE is $(1R, RAA)$



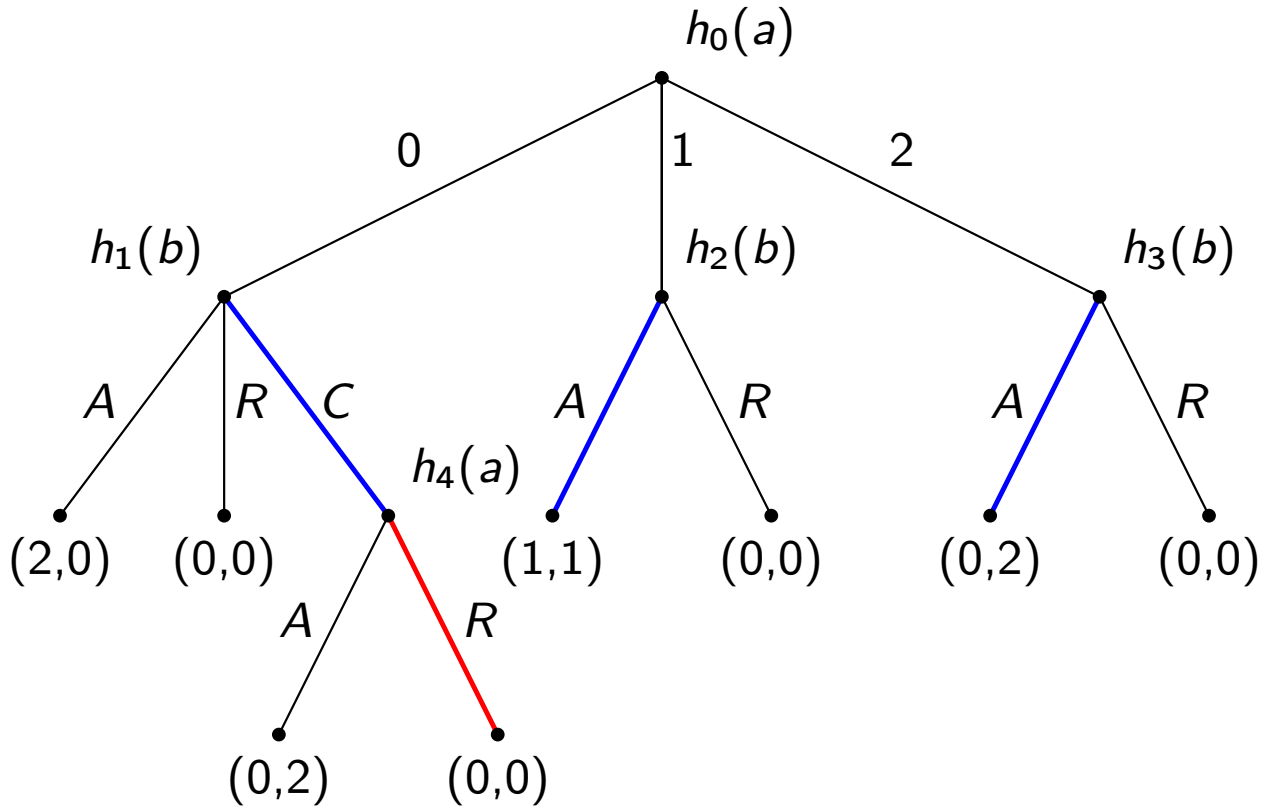
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
   1. Bob chooses R at h_1: The SPE is $(1R, RAA)$
   2. Bob chooses A at h_1:



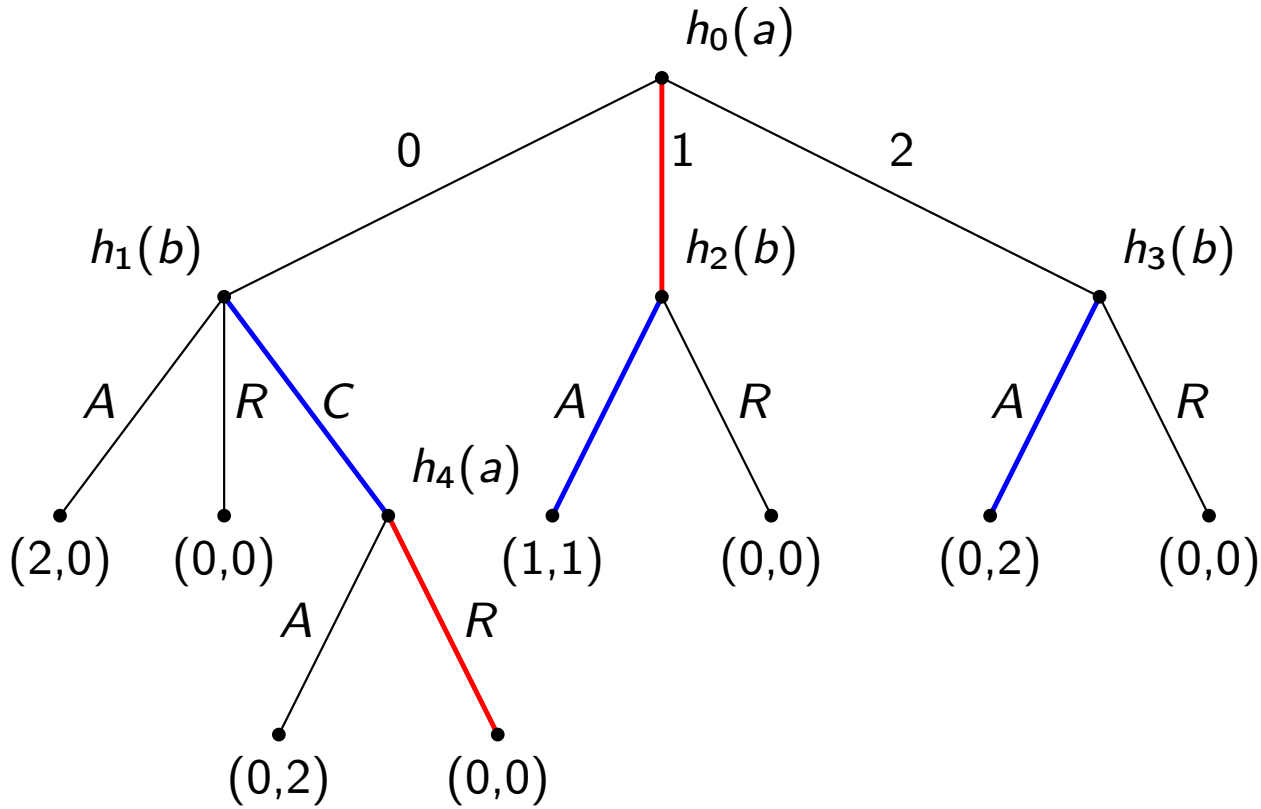
$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
   1. Bob chooses R at h_1: The SPE is $(1R, RAA)$
   2. Bob chooses A at h_1: The SPE is $(0R, AAA)$



$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
   1. Bob chooses R at h_1: The SPE is $(1R, RAA)$
   2. Bob chooses A at h_1: The SPE is $(0R, AAA)$
   3. Bob chooses C at h_1:



$$\begin{aligned} \text{Group 1 nodes} &= \{h_2, h_3, h_4\} \\ \text{Group 2 nodes} &= \{h_1\} \\ \text{Group 3 nodes} &= \{h_0\} \end{aligned}$$

1. Ann chooses A at h_4: The SPE is $(1A, CAA)$
2. Ann chooses R at h_4:
   1. Bob chooses R at h_1: The SPE is $(1R, RAA)$
   2. Bob chooses A at h_1: The SPE is $(0R, AAA)$
   3. Bob chooses C at h_1: The SPE is $(1R, CAA)$

Definition

A strategy profile $s = (s^1, \ldots, s^n)$ satisfies the **one-deviation principle** if for each player i and each h with $P(h) = i$:

$$u^i(s^i, s^{-i}, h) \geq u^i(r^i, s^{-i}, h),$$

for all r^i that agree with s^i everywhere except h.

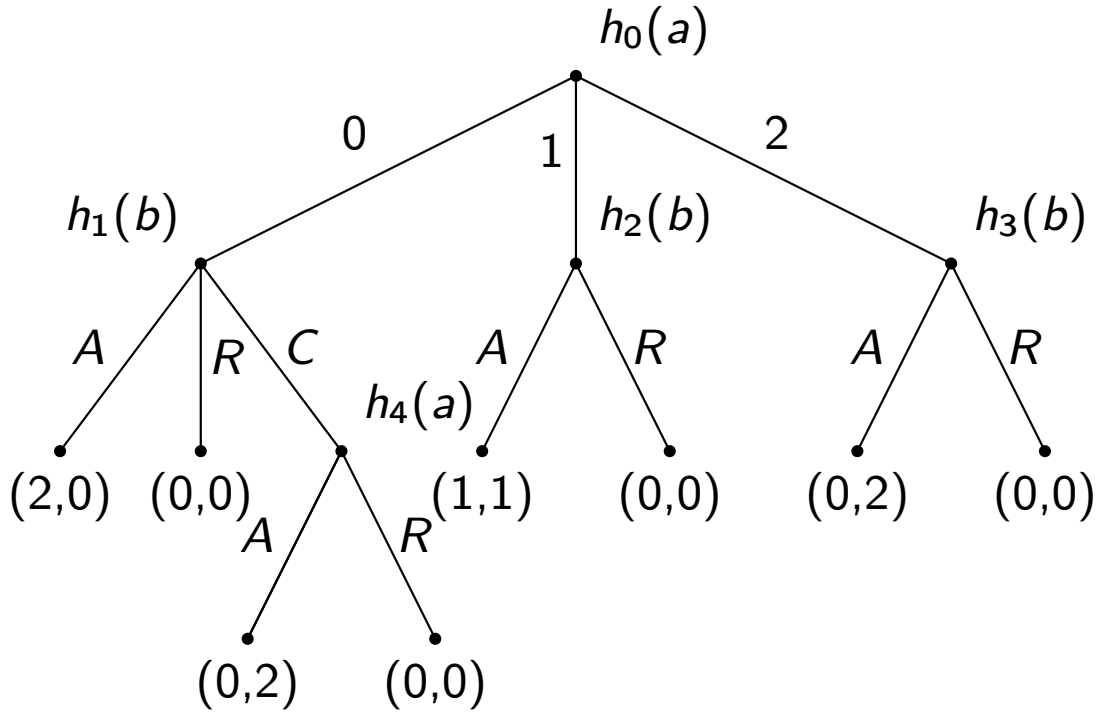
- That is, i does not want to change his action at h.

Theorem

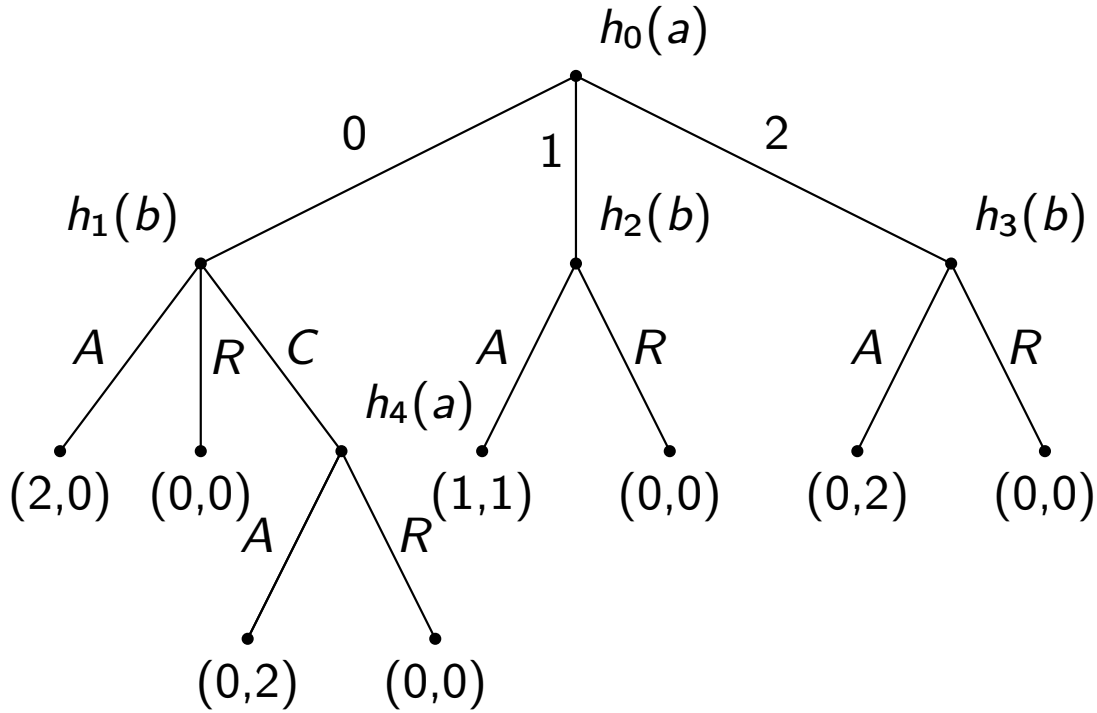
A strategy profile in an extensive game is a SPE

if and only if

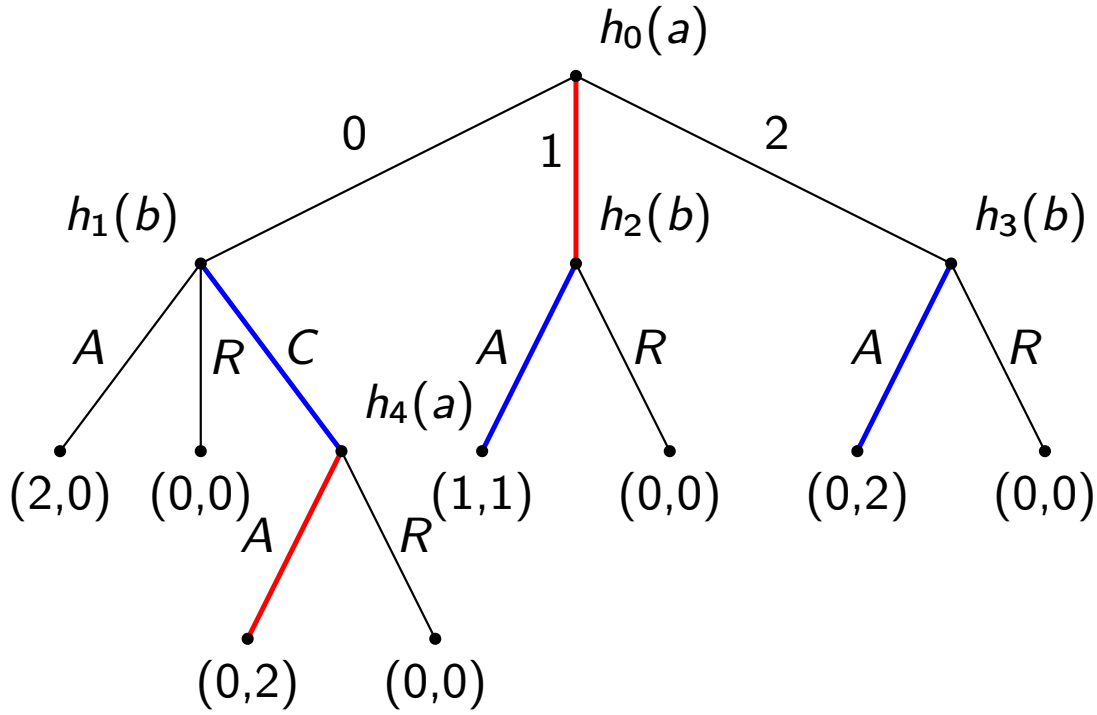
it satisfies the one-deviation principle.



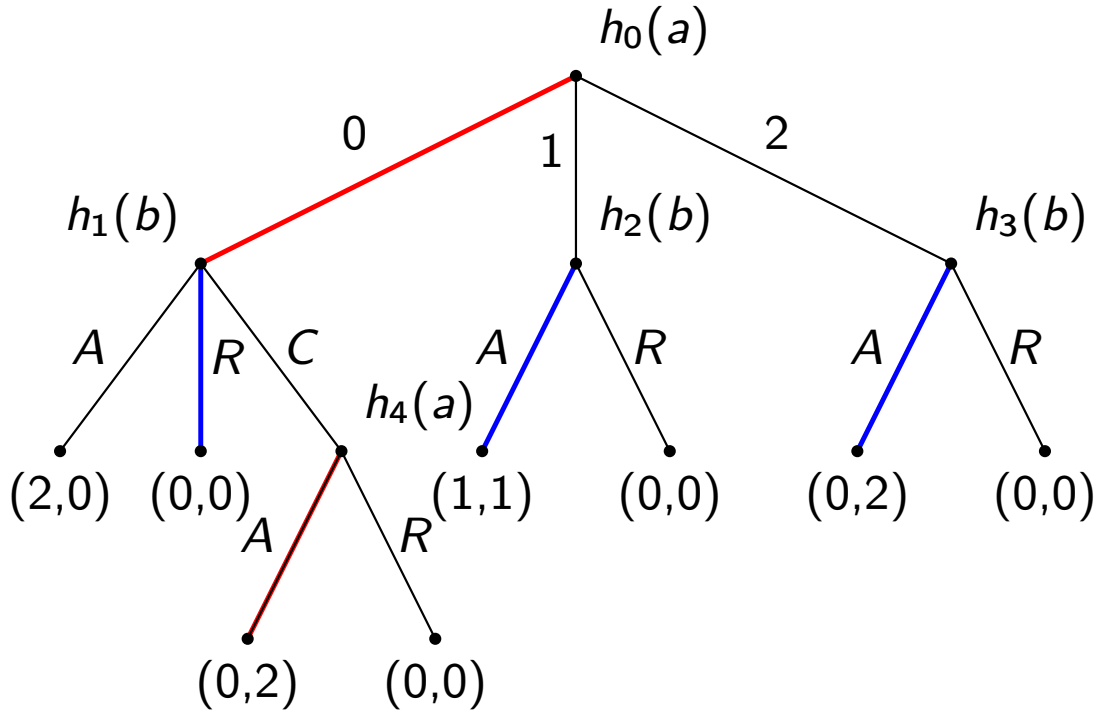
	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0



	AAA	*AAR*	*ARA*	*ARR*	*RAA*	*RAR*	*RRA*	*RRR*	*CAA*	*CAR*	*CRA*	*CRR*
0*A*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,2	0,2	0,2	0,2
0*R*	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
1*A*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
1*R*	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
2*A*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0
2*R*	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0

- At home you need to read Chapter 16 of the textbook Edition 2020) except 16.4 (the part on bargaining) which you can skip.
- Same instructions as last week apply